

Pretty Permutation Parity Proof

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Let p be a permutation of $[1\ 2\ \dots\ n]$. Let $G(p)$ be the number of times in p that a number is greater than a number to its right. For example, $G([2\ 4\ 1\ 3]) = 3$. Note that $G[1\ 2\ \dots\ n] = 0$. A transposition of numbers in adjacent positions changes G by ± 1 . Every transposition can be expressed as a product of an *odd* number of such transpositions. Therefore every transposition changes the parity of G . Thus the number of transpositions used to obtain p is always even or always odd, according as $G(p)$ is even or odd.