November 14, 2018

Errata for *Linear and Geometric Algebra*
Printings 1-4

p. 59, proof of Theorem 4.14N3.
"= |u|^2 + 2u \cdot v + |v|^2" → "≤ |u|^2 + 2u \cdot v + |v|^2".

p. 61, proof of Part (b). “to the base, |b_2|” → “to the base, |b_1|”.

p.62, Problem 4.3.16 should read: \[ |f(b) - f(a)| \leq \sqrt{b - a} \int_a^b (f'(x))^2 \, dx. \]

p. 71, Problem 4.4.4.
Ans. \([0.577, 0.577, 0, 0.577], [0.667, 0, 0.333, -0.667], [0, 0.408, -0.817, -0.408].\)

p. 74, opposite Figure 5.4. “Define \(B_1 + B_2 = (u + v) \wedge w\)” → “Define \(B_1 + B_2\) as shown in the figure”.

p. 76, paragraph before Theorem 5.2. “as bivectors” → “as oriented areas”.

Exercise 5.5. “scalar multiple of \(T\)” → “scalar multiple of \(T_0\)”.

p. 82, sentence following “Note from Part (b)”. And from Part (c): The basis vectors \(\{e_1, e_2, e_3\}\) anticommutate, e.g., \(e_2 e_1 = -e_1 e_2\).

p. 87, last paragraph. “that complex numbers in \(G_3\) and \(G_4\)” → “that complex numbers in \(G^2\) and \(G^3\)”.

p. 100. Additions:
The inner and outer products are coordinate independent (Theorem 6.1-G7).

p. 105, Proof of Theorem 6.15. Sentence should read “Thus Part (c) is proved in this special case.”

Problem 6.3.5 should read: “Complete the proof of Theorem 6.30c”.

p. 115, proof of Eq. (6.20). The proof is not in error but it is much easier to observe that the equation follows from Theorem 6.5 and the definitions of the inner and outer products.

p. 121, bottom. Improve the theorem:

**Theorem.** (Geometric interpretation of \(A \cdot B\).)  
\(A \cdot B = P_B(A)B.\)

b. \(A \cdot B = (-1)^{k(k-1)/2} |B| P_B(A)^* (k = \text{grade}(B)\) and dual taken in \(B\)).

So \(A \cdot B\) is the orthogonal complement in \(B\) of the projection of \(A\) on \(B\), times \((-1)^{k(k-1)/2}|B|\).

**Proof.**  
a. \(A \cdot B = ((A \cdot B)/|B|)B = P_B(A)B.\)

b. Substitute \(B = (-1)^{k(k-1)/2}|B|/(B/|B|)\) (Corollary 6.11b) into Part (a). \(\square\)

p. 161, Exercise 9.9b. “[\(f(b_1)] b_1, b_2\)” → “[\(f(b'_1)] b'_1, b'_2\)”.

p. 164, Exercise 9.13. “Show that \(u\)” → “Show that \(u \neq 0\)”.

p. 168. “From \(e_i(e_i \cdot v) = P_{e_i}\)” → “From \(e_i(e_i \cdot v) = P_{e_i}(v)\)”.
p. 170, Problem 9.4.11. The second paragraph of the statement of the theorem is worded poorly. Replace it with:

Plot the data using the basis $\mathcal{E} = \{e_1, e_2, e_3\}$. See Figure 9.1. Translate the points so they are centered (Problem 4.3.15) at the origin. The dashed arrow in the figure indicates this. We no longer need the original coordinate names for the data so we reuse $(x_1, x_2, x_3)$ for the new coordinates.

Also, reverse the direction of the arrow in the figure.

p. 179, proof of Theorem 9.33. Third line from end: $r = \text{rank}(f) \rightarrow r = \text{rank}(h)$.

p. 185, Problem 9.7.8. Use the polar decomposition.

p. 189, below Exercise 10.9: “By definition ... $O$ on $\mathbb{G}^{n+1,1}$” $\rightarrow$ “By definition ... $O$ on $\mathbb{R}^{n+1,1}$.”

p. 194, statement of Theorem 10.3. $\mathbb{R}^n \rightarrow \mathbb{R}^n$. 
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Printings 1-3

p. 16, should read “it only has to *act* like zero, i.e. it has to satisfy V4. See Exercise 2.1.”

p. 20, (⇐) part of Theorem 2.3’s proof: Replace all four occurrences of U with U.

p. 25, Theorem 2.9, Part (a): Suppose that \( \mathbf{v} \notin \text{span}(\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_r) \).

p. 58, Figure 4.8. Equation in caption should read \( |u+v|^2 = |u|^2 + |v|^2 \).

p. 69, Problem 4.4.8a should read. “If \( \mathbf{u} \neq \mathbf{v} \), then \( d(\mathbf{u}, \mathbf{v}) > 0 \). Also, \( d(\mathbf{u}, \mathbf{u}) = 0 \)”

p. 76, second paragraph. “Bivectors have no shape.” \( \rightarrow \) “Oriented areas have no shape from the standpoint of geometric algebra.”

p. 80, last line: solid \( \rightarrow \) volume.

p. 83, Problem 5.3.4. Add Hint: In Chapter 6 we will learn that vector \( \cdot \) bivector is the vector part of (vector)(bivector).

p. 98, after Definition 6.7. Remove the sentence “So a k-blade is a k-vector, but a k-vector need not be a k-blade.” The third paragraph on p. 99 illustrates the situation.

p. 115, Theorem 6.22. Remove “In particular, \( \mathbf{a} \perp \mathbf{a} \cdot \mathbf{B} \).”

p. 120, NOTE 1. \( \mathbf{v} \rightarrow \mathbf{a} \) everywhere.

p. 124, Theorem 7.1.6. \( \mathbf{A} || = a_1 \parallel \wedge a_2 \parallel \wedge \cdots \wedge a_j \parallel \rightarrow \mathbf{A} || = a_1 \parallel \wedge a_2 \parallel \wedge \cdots \wedge a_j \parallel \), “take \( i = 2 \)” \( \rightarrow \) “take \( j = 2 \)”, “\( P_B(\mathbf{A}) \leq |\mathbf{A}| \) follows” \( \rightarrow \) “\( |P_B(\mathbf{A})| \leq |\mathbf{A}| \) follows”.

p. 125, Fig. 7.8. I have had a report from a reader that the planes in the figure are not seen in his book. I am not aware of other instances of this. Figures 7.2, 7.8, and 7.10 are reproduced below.

p. 144, proof of Theorem 8.9. Theorem 4.19 \( \rightarrow \) Theorem 4.18.
p. 163, new part of Problem 9.2.7:
c. Use Problem 7.1.4 to show that only 1 and 0 can be eigenvalues of a
projection.

p. 164, Problem 9.2.16c. Theorem 9.3 → Corollary 9.3.

p. 175, Theorem 9.16(iv). Replace (incorrect) proof:
From the proof of Lemma 9.27 we have the situation of Figure 5.5.5. The
figure shows that \( f \) is a reflection through the 1-dimensional subspace of scalar
multiples of \( v - u \). So this case reduces to Case (ii).

p. 191, change to: If you check, you will find that it is true for \( n = 1, 2, 3, \ldots, 40 \). However, don’t jump to conclusions: for \( n = 41 \), \( (41)^2 - 41 + 41 = 41^2 \), which is not a prime. Despite the 40 examples where the proposition is
ture, the example \( n = 41 \) is sufficient to disprove the proposed theorem.

p. 194, second paragraph: We also use the notation \( gf = g \circ f \).


p. 207, Index entry: orthogonal complement 66, 106
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Printings 1-2,
and new problems

p. 3, paragraph 3 (improved wording). Parallel lengths with the same norm
and orientation are different geometric objects. Nevertheless, we will consider
them equal. In other words, we ignore their position.

p. 22, Definition 2.5. Add at the end (for clarity): “We say that $V$ spans
$\text{span}(V)$.”

p. 39, second paragraph after Definition 3.4. Eq. (3.2)) $\rightarrow$ (Eq. (3.3).

p. 54, Exercise 4.6. Ans. $\cos \theta = 32/(14 \cdot 77)^{\frac{1}{2}}$.

p. 54, Exercise 4.7. Ans. $\frac{32}{17}(4e_1 + 5e_2 + 6e_3)$.

p. 65, Part (b). The parenthesis in “(We can also . . . ” should close at the
end of the paragraph.

p. 73, Section 5.1, paragraph 4 (improved wording). Recall that parallel
lengths with the same norm and orientation are considered equal. Similarly,
parallel areas (i.e., areas in nonintersecting planes) with the same norm and
orientation are considered equal. In other words, we ignore their shape and
position. Thus $B$ in the figure is equal to the interior of a parallelogram with
the same norm and orientation in a parallel plane.

p. 75, replace last two sentences of Note 1. The sum $B_1 + B_2$ of nonzero
oriented areas in the two planes does not represent an oriented area.

p. 75, remove Note 3.

p. 79, paragraph 4 (improved wording). Volumes with the same orientation
and norm are considered equal. In other words, we ignore their shape and
position.

p. 88, Problem 5.4.4. “three” $\rightarrow$ “first” and remove “they satisfy”. Replace
the last paragraph with “Using the full series shows that $e^{i\theta} = \cos \theta + i \sin \theta.”$

p. 90, Figure 5.19. $-(2\pi - \theta) \rightarrow 2\pi - \theta$.
Exercise 5.26. $R_{i(\theta-2\pi)}(u) = R_{i\theta}(u) \rightarrow R_{i(\theta-2\pi)}(u) = R_{i\theta}(u)$

p. 92, new Problem 5.5.9. Prove Rodrigues’ formula for a rotation around
axis $n$ ($|n| = 1$) by scalar angle $\theta$:

$$R_{(n,\theta)}x = \cos \theta x + \sin \theta(n \times x) + (1 - \cos \theta)(n \cdot x)n.$$ 

p. 93, second sentence. “Theorem 1.2” $\rightarrow$ “Theorem 1.8”.

p. 98. Add immediately after Definition 6.16 for clarity:
“(So a $k$-blade is a $k$-vector, but a $k$-vector need not be a $k$-blade.)”
p. 101, end. $\sum_{j,k=0}^{n} a_j b_k \rightarrow \sum_{j,k=1}^{n} a_j b_k$.

p. 103, Eq. (6.12). “$k - j$” $\rightarrow$ “$j + k$”


p. 109, Problem 6.5.2. Problem 6.1.1 $\rightarrow$ Definition 6.9.

p. 122, Theorem 7.4c proof. “we need to verify Part (d)” $\rightarrow$ “we need to verify Part (c)”.

p. 122, Theorem 7.4 proof, last line. “Exercise 6.1.2” $\rightarrow$ “Problem 6.1.2”.

p. 125, under Composition of rotations. Should read $Z_1 = e^{-i_1 \theta_1/2}$ and $Z_2 = e^{-i_2 \theta_2/2}$.

p. 131, Problem 7.3.2. The answer is $e_1 + 3e_3$.

p. 131, Problem 7.3.9a. Change the hint to: Use Theorem 7.9.

p. 131, last paragraph. “if we imagine” $\rightarrow$ “in the special case”.


p. 144, Exercise 8.16. “Eq. (8.7)” $\rightarrow$ “Eq. (8.8)”.

p. 152, Problem 8.4.3. “a vector space $\mathbf{V}$” $\rightarrow$ “$\mathbb{R}^n$”.

p. 158, Problem 9.1.8. Add to the hypotheses that $f$ is one-to-one.

p. 158, Problem 9.1.9. Problem should read: Find a formula similar to Eq. (9.4) for bases which are not orthonormal. Use reciprocal bases (Problem 6.6.8).

“Choose the smallest $m$ so that the $m$ vectors . . . .” $\rightarrow$
“Choose the smallest $m$ so that the $m + 1$ vectors . . . .”

p. 167, lines -5 and -6:

$$f^2(u) + bf(u) + ci(u) = (f^2 + bf + ci)(u) = (f^2 + bf + ci)q(f)(v) = p(f)(v) = 0.$$  

p. 168, Problem 9.3.3c. “$Hint$: Start with $K(f|_B) = \{0\}$. “ $\rightarrow$
“$Hint$: Start with $K(f|_B) = \{0\}$. “

p. 174, Lemma 9.27. Add “or reflection” to the end of the statement of the lemma.

p. 175, below Eq. (9.17):
“According to Lemma 9.27, there are three possibilities:” $\rightarrow$
“According to Lemma 9.27, there are four possibilities:”

p. 175, Theorem 9.16. Add to proof:
(iv) $f$ has an invariant 2-dimensional subspace on which it is a reflection. By Lemma 9.27 and Problem 5.5.5 this is a composition of Cases (i) and (ii), and so reduces to them.

p. 184, Problem 9.7.1b. The matrix $A$ must be square.

p. 198, top. `numpy.set_printoptions(precision=3)` → `set_printoptions(precision=3)`

p. 200, line 5. Change to “$B.exp()$ e$^B$”.
Errata for *Linear and Geometric Algebra*

Printing 1

p. vii, footnote. “available the” → “available at the”.

p. viii, line 22. “definition a determinant” → “definition of a determinant”.

p. 8, line 10. “Ordered triples \((x, y, z)\)” → “Ordered pairs \((x, y, z)\)”.

p. 8, Theorem 1.3c. “[0] = (0, 0, 0)” → “[0] = (0, 0)”.

p. 13, Theorem 1.8, line 4. “\([0] = (0, 0, 0)\)” → “\([0] = (0, 0)\)”.

p. 19, Problem 2.1.6a. Change to “Let \(a(x, y) = (ay, ax)\) be the usual scalar multiplication on pairs \((x, y)\). Define \((x_1, y_1) + (x_2, y_2) = (0, 0)\). Does this define a vector space?”

p. 20, Theorem 2.3 statement. “Let \(U\) set” → “Let \(U\) be a set”.

p. 25, Theorem 2.9b, proof. Should read “Since the vectors \(v_1, \ldots, v_r\) are linearly independent.”

p. 30, Theorem 2.20b. Add “for \(V\)” at the end of the statement.

p. 32, -12 (up from the bottom), “all vector spaces” → “all finite dimensional vector spaces”.

p 35, Eq. (3.5) should read:

\[
n \begin{bmatrix} m \\ \\ m \end{bmatrix} = \begin{bmatrix} n \end{bmatrix} \quad \text{(Mnemonically: } (n \times m) \times (m \times 1) = n \times 1.\text{)}
\]

p. 36, line 2: \((n \times m) \times (m \times p) = n \times p\).

p. 36, line 5: “n-dimensional” → “m-dimensional”.

p. 36, Eq. (3.9). “\(a_{in} b_{nk}\)” → “\(a_{im} b_{mk}\)”.

p. 49, Problem 3.2.2c. \(b \in \mathbb{R}^3\) → “\(b \in \mathbb{R}^2\)”.

p. 49, Problem 3.2.5. Change to “Suppose \(A\) is an \(n \times n\) matrix with \(A x = 0\) for some \(x \neq 0\). Is there a unique solution to \(A x = b\) for every \(b\)? Explain.”

p. 56. Problem 4.2.2a. Change to read “Describe the entries of the \(k \times k\) matrix \(A^* A\) in terms of the \(v\)’s.”
p. 58, Equation above Fig. 4.7 should read \(|\mathbf{u} + \mathbf{v}|^2 = |\mathbf{u}|^2 + |\mathbf{v}|^2 - 2|\mathbf{u}||\mathbf{v}| \cos \theta\) as in the figure caption.

p. 59. Last line of proof of Theorem 4.14: “Schwartz” → “Schwarz”.

p. 62, Problem 4.3.14a should reference Theorem 3.2.

p. 64, prior to Eq. (4.15): Now subtract from \(\mathbf{u}_2\) its projection on \(\mathbf{b}_1\) ...

p. 65, line 1: ... its projections on \(\mathbf{b}_1\) and \(\mathbf{b}_2\) ...

p. 67, Eq. (4.18). “\((\mathbf{v} \cdot \mathbf{e}_{i+1}) \mathbf{e}_{i+1}\)” → “\((\mathbf{v} \cdot \mathbf{e}_{r+1}) \mathbf{e}_{r+1}\)”.

p. 68, bottom. Here is a non-Java least squares demo: http://demonstrations.wolfram.com/LeastSquaresCriteriaForTheLeastSquaresRegressionLine/

p. 69, Problem 4.4.3. The basis \(\mathbf{e}_i\) must be orthonormal.

p. 75, Note 3. The last sentence should read “Moreover, a clockwise orientation viewed from one side is counterclockwise when viewed from the other.”

p. 76, end of first paragraph: “Its orientation is given by the way \(\mathbf{u}\), the first vector in the product \(\mathbf{u} \wedge \mathbf{v}\), points along the boundary.”

p. 79. The paragraph starting with “Three oriented lengths placed tail-to-tail determine an oriented volume \(\mathbf{u} \wedge \mathbf{v} \wedge \mathbf{w}\), their outer product.” should continue “The trivectors \(\mathbf{u} \wedge \mathbf{v} \wedge \mathbf{w}\) and \(-\mathbf{u} \wedge \mathbf{v} \wedge \mathbf{w}\) have opposite orientations.”

p. 84, paragraph following Exercise 5.13. Improved:

A unit pseudoscalar \(\mathbf{i}\) determines exactly one plane. Thus “the plane \(\mathbf{i}\)” can serve as an abbreviation for “the plane with unit pseudoscalar \(\mathbf{i}\)”. This usage is similar to the abbreviation “the point \((2, 4)\)” for “the point with coordinates \((2, 4)\)”. These are useful shortenings, but please be clear that oriented planes and points are geometric objects, whereas \(\mathbf{i}\) and \((2, 4)\) are their mathematical representations.

p. 84, Exercise 5.14a, first sentence: “The square of a real scalar cannot be negative.”

p. 85, Definition 5.16, should read: “... is a member of \(\mathbb{G}^n\) ... ”

p. 88, Problem 5.4.1, last \(\theta_1\) should be \(\theta_2\).

p. 91, Eq. (5.18) should read \(\cos 60^\circ = \frac{\mathbf{e}_1 - \mathbf{e}_2 + \mathbf{e}_3}{\sqrt{3}} \mathbf{I} \sin 60^\circ\).

p. 92, Problem 5.5.6, should read: Verify Step (3) in Eq. (5.14).

p. 98, very end: Theorem 5.15 → Theorem 5.24.

p. 104, Problem 6.3.3, change to “Show that \(\mathbf{u}^2 \mathbf{v}^2 = (\mathbf{u} \cdot \mathbf{v})^2 - (\mathbf{u} \wedge \mathbf{v})^2\) by factoring the right side.”

p. 105, line 8: for → form.

p. 111, Theorem 6.28, first sentence should read: "Let \(\mathbf{a}\) be a vector and \(\mathbf{B}\) a \(k\)-blade.”
p. 113, second line of proof of Theorem 6.30 should read: “$\mathbf{A} \neq \mathbf{a}_1 \cdots \mathbf{a}_{j-1} \mathbf{c}$”.

p. 115, In reciprocal basis problem:
$$\mathbf{b}_i = (-1)^{i-1}(\mathbf{b}_1 \wedge \mathbf{b}_2 \wedge \cdots \wedge \mathbf{b}_i \wedge \cdots \wedge \mathbf{b}_n)/(\mathbf{b}_1 \wedge \mathbf{b}_2 \wedge \cdots \wedge \mathbf{b}_n).$$

p. 115, Problem 6.6.8f, should read: Use the expansions $\mathbf{u} = \sum_i u_i \mathbf{b}_i$ and $\mathbf{v} = \sum_j v_j \mathbf{b}_j$ to obtain a formula for $\mathbf{u} \cdot \mathbf{v}$.

p. 115, last sentence of footnote: “This book does not take it up.” → “This book does not take it up.”

p. 116, Theorem 6.19b: “is volume” → “is the volume”.

p. 119, Proof of Theorem 7.1: “Theorem 6.15” → “Eq. (4.17)”.

p. 120, projection definition. “is the blade” → “is”.
(The projection might be 0.)

p. 121, Theorem 7.3. Strike “the blade”. (The projection might be zero.)

p. 126, Eq. (7.11). Elaboration of justification of Step (2): Separate $\mathbf{M} \wedge \mathbf{N}$ into its parts, and then use Parts (a) and (d).


p. 129, line 14. “depend the way” → “depend on the way”.

p. 131, Problem 7.3.9c. All “$R$”s → “$F$”s.

p. 138, Exercise 8.5a. $f(\mathbf{u}) \rightarrow f(\mathbf{u})$.

p. 141, “Below $\mathcal{R}(f)$ ... remaining vectors in $\mathbf{V}$” → “below $\mathbf{0}$ are the remaining vectors in $\mathbf{V}$.”

p. 146, line 8. “less than of” → “less than or”.

p. 146, Theorem 8.14b. “$\mathcal{K}(f)\perp$” → “$\mathcal{K}(A)\perp$”.

p. 147, Problem 8.2.5.a. “$n - r$” → “$m - r$”.

p. 150, Problem 8.3.2. “Now adapt Eq. 7.9 ... .” → “Now adapt the proof of Theorem 7.4c.”

p. 153, line 10. “representation an an” → “representation as an”

p. 157, following Theorem 9.9. “$i^{th}$ column” → “$i^{th}$ row”.

p. 157, Theorem 9.10. Change Parts (a)-(c) to refer to columns and Part (d) to refer to rows.

p. 158, Problem 9.1.11. “For Part (e)” → “For Part (d)”.

p. 160. Delete the first sentence.

p. 163, Problem 9.2.8. $M_B$ → “$F_B$”.

p. 170. “$-(a + d) \pm \sqrt{(a - d)^2 + 4b^2}$” → “$a - d \pm \sqrt{(a - d)^2 + 4b^2}$.”