March 12, 2019

Errata for *Vector and Geometric Calculus*
Printings 1-4

Note: “p. m (n)” refers to page m of Printing 4 and page n of Printings 1-3.

p. 30, proof of Theorem 3.8. “the last sum in Eq (3.6) approaches 0.” → “the right side of Step (3), divided by |h|, approaches 0 with h.”

p. 31 (29), just before Theorem 3.10. $f'_x(h) = [f'_x(h)] 
→ \[ f'_x(h) \] = \[ f'_x \][h].$


p. 44, bottom. “for $y(x)$ near $a$” → “for $y$ in terms of $x$ near $a$”.

p. 56 (54), an omission, not an error. New second paragraph after Definition 4.5: “The tangent space is a vector space (LAGA Exercise 8.12).”

p. 58 (55), caption of Figure 5.10. “$f'$ maps” → “$f'_p$ maps”.

p. 62 (58), Definition 5.2. Add a footnote after the first line: “i.e., the scalar coefficients of $F$ are differentiable.”

p. 64, Problem 5.2.10. Replace “$f' x(a)$” with “$f'_x(a)$”.

p. 79 (73), Problem 5.4.1a. “Eq. (6.25)” → “Eq. (5.19)”.

p. 85, first line of the proof of Theorem 6.5, improved: “First, $\partial_i f(x)h_i = \nabla f(x) \cdot h = 0$.”

p. 86, Definition 6.3. In “a strict local minimum at $x$ if $f(x) \leq f(x + h)$” change to “$f(x) < f(x + h)$”.

p. 96, Exercise 7.1. The approximate value of the integral is 0.2570120954.

p. 99, Problem 7.1.1. At end: “Parts (c)-(d)”.

p. 105. Remove Problem 7.2.9. It is a duplicate of Problem 7.2.3.

p. 115, Problem 7.2.3. Integral should read $\int_{[a,b]} \sqrt{r'^2 + r^2} \, d\theta$.

p. 129, just before Figures 8.2-8.4. Strike “to a scalar”.

p. 131, Problem 9.1.4. Displayed equation should read $2\pi \int_{a}^{b} \sqrt{1 + r'(z)^2} \, r(z) \, dz$.

p. 132. The formula for area in the middle of the page should be $\int_A |x_u(u,v) \wedge x_v(u,v)| \, dA$.

p. 138, Problem 9.2.2. The answer is $-4\pi/3$.

p. 142, last paragraph, first sentence. “multiple integrals” → “directed integrals”.

p. 144, Exercise 10.2. Replace the hint with “Fact: The boundary of a boundary is empty: $\partial(\partial M) = \emptyset$.”

p. 147, improved version of
**Corollary 10.3** (Generalized divergence theorem). Let $M$ be a bounded $m$-dimensional manifold in $\mathbb{R}^m$ and $\mathbf{n}$ be its outward normal. Let $F$ be a multivector field on $M$. Set $d\sigma = \mathbf{n} d^{m-1} x$. Then

$$\int_M \nabla \cdot F \, d^m x = \oint_{\partial M} d\sigma \cdot F.$$ 

*Proof.* Suppose first that $F$ is of a single grade, $g$. Use the extended fundamental identity (LAGA Theorem 6.28) to split the geometric product in both integrands of Eq. (10.5) into inner and outer products, of grades $g-1$ and $g+1$ respectively. Equating the inner product parts gives Eq. (10.7) for a single grade $F$.

To finish, apply this to each grade of a multigrade $F$ and add. $\square$

p. 153, first displayed equation: \((\nabla \times f)(x^*) \cdot \mathbf{n} \rightarrow ((\nabla \times f) \cdot \mathbf{n})(x^*)\). And the left side of Eq. (10.14) should read: \(((\nabla \times f) \cdot \mathbf{n})(x_0)\).

p. 166, Theorem 11.8. “Let $x(u(t), v(t))$” $\rightarrow$ “Let $x(u_1(t), u_2(t))$.”
Errata for Vector and Geometric Calculus
Printings 1-3

p. 30, Theorem 3.10, first line of proof:
\((\partial_{i_1} f_1(x), \ldots, \partial_{i_n} f_n(x)) \rightarrow (\partial_{i_1} f_1(x), \ldots, \partial_{i_m} f_m(x))\).

p. 40, just before the problems. “series of a vector valued function f centered” \(\rightarrow\) “series of an f centered”.

p. 44, first line. “differentiable” \(\rightarrow\) “continuously differentiable”

p. 51, Theorem 4.3 should read:
Let \(x_1(t)\) and \(x_2(t)\) parameterize curves and suppose that \(x_1'(t)\) and \(x_2'(t)\) exist. Then \((x_1(t)x_2(t))'\) exists and
\[(x_1(t)x_2(t))' = x_1'(t)x_2(t) + x_1(t)x_2'(t).\]
Change Eq. (4.6) similarly. Remove the paragraph following the theorem.

p. 55, change bottom to
Let \(f\) be a 1-1 map between manifolds of equal dimension. Then the tangent map \(f'_p\) maps \(T_p\) to \(T_{f(p)}\).

p. 58. Remove Problem 4.3.6 Part (b) and restate Part (a):
The definition of the determinant of a linear transformation on a vector space (LAGA Definition 8.21) does not apply to the linear transformation \(f'_p\) because it is between two vector spaces. Define \(\text{det}(f'_p)\). \(Hint:\) See LAGA Definition 8.21.

p. 64, Problem 5.2.11. \(\nabla \wedge e = -\partial_t B \rightarrow \nabla \wedge e = \partial_t B.\)

p. 76, Problem 11.2.8. New Part (b): Define a directional derivative for fields defined on a surface by \(\partial_{\mathbf{t}} f(p) = (\mathbf{h} \cdot \partial) f(p)\) (Definition 11.10). Compute \(\partial_{\mathbf{t}} \mathbf{t}\) on the equator. \(Ans. -\sin \theta i + \cos \theta j)/\rho.\)
Note that at the equator \(\mathbf{t}\) is in the tangent plane but \(\partial_{\mathbf{t}} \mathbf{t}\) is not.

p. 82, Theorem 6.7 statement. \(g(x_0) = c \rightarrow g(x) = c.\)

p. 105, Problem 7.4.3. Field should be \(e^x(\sin(xy) + y \cos(xy))i + xe^x \cos(xy)j\)

p. 116, Exercise 8.8. The answer is \(\pi(e^4 - 1).\)

p. 132, Corollary 10.3. “Let \(f\) be a multivector field” \(\rightarrow\) “Let \(f\) be a vector field”

p. 134, Problem 10.2.6c. \(\int_C e \cdot ds = \partial_t \int_S B \cdot dS \rightarrow \int_C e \cdot ds = -\partial_t \int_S B \cdot dS\)

p. 186. Add “65” and “66” to gradient entry. Add “98” to curl entry. Add “124” to divergence entry.
Errata for *Vector and Geometric Calculus*  
Printings 1-2

p. 14, line 8: Delete “in”.

p. 23, line 4: Change the period after “tangents” into a comma.

p. 28, Exercise 3.9: Change “Eq. (3.23)” to “Eq. (3.18)”.

p. 30, Exercise 3.13. \( f: U \subseteq \mathbb{R}^n \to \mathbb{R}^m \to f: \mathbb{R}^n \to \mathbb{R}^m \).

p. 38, Problem 3.4.12c. Change to “The ideal gas law is \( pv = nRT \), where \( n, p, v, T \) are the number of moles, pressure . . . .”

p. 43, last two lines. \( f(a, b) = 0 \to f(a, b) = (2, 2) \).

p. 55, Figure 5.10, caption. “onto \( S \)” → “to \( S \)”.

p. 59, Definition 4.1. “Let \( q \in A \) and set \( p = x(q) \).” → “Let \( t \in A \) and set \( p = x(t) \).”

p. 59, line -5: Change “Eq. (5.2) and Eq. (3.6)” to “Eq. (5.2) and Eq. (3.23)”.

p. 64, bottom. Problem 4.3.12 → LAGA Problem 4.3.12

p. 69, Eq. (5.17). \( \partial x_i \partial w_j e_i \to \partial x_i \partial w_j e_i \).

p. 72, Exercise 5.23. Equations should read \( \hat{\phi} = \cos \phi (\cos \theta i + \sin \theta j) - \sin \phi k, \quad \hat{\theta} = -\sin \theta i + \cos \theta j \).

p. 75, footnote. \( \mathbb{R}^3 \to \mathbb{R}^n \).

p. 76, Problem 5.5.1b, second printing only. “even though \( \hat{t} \) is” → “even though \( \hat{t} \) is”.

p. 79, Theorem 6.5. A much better proof: 
**Proof.** Since \( \nabla f(x) = 0 \), \( \partial f(x)h_i = 0 \). And \( \partial f(x)h_i h_j > 0 \) for \( h \neq 0 \), since \( Hf(x) \) is positive definite. Then \( \partial f(x + t*h)h_i h_j > 0 \) for small \( t*h \neq 0 \), since the partial derivatives are continuous at \( x \). The theorem now follows from Eq. 3.2. \( \square \)

p. 81, Problem 6.1.2c. \( \lim_{(x, y) \to \infty} \to \lim_{(x, y) \to \infty} f(x, y) \).

p. 90, equation mid-page, \( \int_{[a, b]} af dx = a \int_{[a, b]} f dx \to \int_{[a, b]} cf dx = c \int_{[a, b]} f dx \).

p. 91, after the sentence beginning with Think of. “Divide \( C \) into infinitesimal parts. Multiply the value of \( F \) on each part by the infinitesimal length \( ds \) of the part. Add to form the integral.”

p. 96, note at the bottom of the page. “here here” → “here”.

p. 98, Problem 7.3.4c. Misplaced “): “\( F(x(u, v) x_u(u, v)) \)” → “\( F(x(u, v)) x_u(u, v) \)”.


p. 100, proof of Theorem 7.13: ‘it independent” → “it is independent”.
p. 101, Exercise 7.24b. “not conservative” → “not conservative in $\mathbb{R}^2 - \{0\}$$\)".

p. 102, following Definition 7.16. “All simple closed curves” → “All closed curves”.

p. 103, line -7. Switch “m” and “M”.

p. 103, line -5: Change “Eq. (7.10)” to “(Eq. (7.14))’”.

p. 107, line -3: Change “set open” to “open set”.

p. 112, Exercise 8.2. $\int_{y=0}^{1} \rightarrow \int_{y=0}^{2}$.

p. 124, Problem 9.2.3: “scalar + trivector” → “vector + trivector”.

p. 131, Fig. 10.6: Arrows should be reversed, as the $M_i$ are “oriented clockwise”.

p. 136, second line of the proof of Corollary 10.5: $(-1)^{2\times2} → (-1)^{2\times1}$.

p. 142, below Corollary 10.10. \(f(x) = \int_a^x f'(t)dt + f(a)$.

p. 146, lines 12 and 17: Change “Theorem 4.3b” and “Theorem 4.3” (both) to “Eq. (4.6)”.

p. 148, Theorem 11.5, Proof. $\mathbf{\bar{x}} \rightarrow \mathbf{\dot{x}}$, twice

p. 151, line above Def 11.10: Change “Definition 5.23” to “Definition 5.15”.

p. 158, middle displayed line in the proof of Theorem 11.25. Drop the middle term.


p. 162, line 5. Delete “is”. 
Errata for *Vector and Geometric Calculus*  
Printing 1

Due to publisher error the shading in several of figures is washed out in some copies of the book. The correct shading is shown below. I think that if Figure 2.1 is OK in your book, then all figures are.

p. 4, Exercise 1.1. “$c_{ij} \rightarrow c_{ik}$”.

p. 6, Problem 1.1.1. “$x(t) \rightarrow x(\theta)$”.
p. 8, Exercise 1.11. Delete “We will do this often.” Add “This will allow us to specialize formulas for surfaces defined parametrically to surfaces defined by \( z = f(x,y) \). Exercise 5.25 is an example.”

p. 11, Exercise 1.14. “\( x \neq 0 \)” → “\( x > 0 \)”.

\[ \phi = \arccos(z/r) \rightarrow \phi = \arctan(r/z). \]

p. 22, bottom. Remove “The definition shows that \( \partial_i F \) has the same grades as \( F \).” Parts disappear if their partial derivative is zero.

p. 23, Exercise 3.1. “\( \text{RE} \rightarrow \text{R} \)” → “\( \text{R} \rightarrow \text{R} \)”.

p. 33, Problem 3.2.1. Append the sentence “Then for fixed \( x \), the differential is the linear transformation \( h \mapsto f'(x)h \).”

p. 33, Problem 3.2.3. \((\rho,\theta,\phi) \rightarrow (\rho,\phi,\theta)\)

p. 34, first and second displayed equations should read
\[
\begin{align*}
x, h \in \mathbb{R}^n \Rightarrow (g \circ f)(x) \in \mathbb{R}^p \Rightarrow (g \circ f)'(x)(h) \in \mathbb{R}^p, \\
x, h \in \mathbb{R}^n \Rightarrow f'_x(h) \in \mathbb{R}^m \Rightarrow (g_{f(x)} \circ f'_x)(h) \in \mathbb{R}^p.
\end{align*}
\]

p. 34. Line should read \( \frac{4}{...} + [g'(R(h)) \mid h] + S(kh)|kh| \).

p. 34. Replace the end of the page with the following:
The added phrase “divided by \(|h|\)” is the reason for the changes.
To finish, we show that the term in brackets above, divided by \(|h|\), approaches zero with \(|h|\). First, using the continuity of \( g' \) (Theorem 2.10),
\[
\lim_{h \to 0} g'(R(h)) = g'(\lim_{h \to 0} R(h)) = g'(0) = 0.
\]
Second, with \(|f'|_O\) the operator norm of \( f' \),
\[
|kh| \leq |f'(h)| + |R(h)||h| \leq |f'|_O|h| + |R(h)||h|.
\]
Thus, since \( (h \to 0) \Rightarrow (kh \to 0) \Rightarrow (S(kh) \to 0), \lim_{h \to 0} |S(kh)|/|kh|/|h| = 0. \)

p. 37, statement of Theorem 3.16.
“Then the inverse function \((f'_n)^{-1}\) → “Then the inverse function \( f^{-1} \).”

p. 40, following Theorem 3.13: “In other words, \( \partial_h f \) is linear in both \( h \) and \( f \).”

p. 40, Problem 3.3.5. The variable names I used lead to confusion. Change to \( f(x,y) = (x \cos y, x \sin y) \). And add “(All coordinates are cartesian.)”

p. 40, Problem 3.3.3. “continuity of \( f \) at \( x \)” → “continuity of \( f \) at \( x \).”

p. 41, Statement of Theorem 3.19.
“has a differentiable inverse” → “has a continuously differentiable inverse”.

p. 42, third line. Remove “there is a neighborhood of each \( y_i \) in which”.

p. 45, Exercise 3.6.1b. “Determine \( \partial \rho/\partial x \).”

p. 48, below Eq. (4.3). “of higher dimension” → “in higher dimensions”.
Equation (4.4) in Section 4.1 established the notation of $m$-dimensional manifolds $M$ as subsets of $\mathbb{R}^n$. However, Sections 4.2 and 4.3, while mostly internally consistent, are inconsistent with this notation. The following changes remove the inconsistency:

- p. 49, 2nd paragraph: "a curve $C$ in $\mathbb{R}^n."$
- p. 50, Theorem 4.2: $m$ to $n$.
- p. 51, Theorem 4.4: $\mathbb{R}^m$ to $\mathbb{R}^n$.
- p. 52, Theorem: $\mathbb{R}^m$ to $\mathbb{R}^n$, $\mathbb{R}^n$ to $\mathbb{R}^n$.
- p. 53, First sentence and left part of Figure 4.5: $\mathbb{R}^m$ to $\mathbb{R}^n$.
- p. 53, Definition 4.5: $\mathbb{R}^m$ to $\mathbb{R}^n$.
- p. 53, line following Eq. (4.11): $m$ to $n$.
- p. 55, Theorem: $\mathbb{R}^m$ to $\mathbb{R}^n$, $\mathbb{R}^n$ to $\mathbb{R}^n$.

p. 49, Paragraph 2. "Let $x(t)$ parameterize a curve $C$. Then $x$ has a nonzero differential (p. 49). By Problem 3.2.1 it is . . . “

p. 50, line after Theorem 4.3. “Definition 4.5” $\rightarrow$ “Eq. (4.5)"

p. 52, Theorem. “$f'_p$ is one-to-one.” $\rightarrow$ “$f'_p$ restricted to $T_p$ is one-to-one.”

p. 52, Problem 4.2.4.
  a. Show that components of $\Omega$ must have grade 2 or 3.
  b. Show that components of $\Omega$ must have grade 0 or 2.

p. 53, Eq. (4.10). “$\lim_{h \to 0}$” $\rightarrow$ “$\lim_{h \to 0}$”

p. 53, sentence below Definition 4.5. “Recall that the differential $x'_q$ is one-to-one and maps linearly independent vectors ...

p. 55, Theorem. “$f'_p$ is one-to-one.” $\rightarrow$ “$f'_p$ restricted to $T_p$ is one-to-one.”

p. 56, Problem 4.3.1b. $x_u \wedge x_v \rightarrow x_\phi \wedge x_\theta$.

p. 57, Definition 5.1. “Let $M$ be a manifold in $\mathbb{R}^n$. A field on $M$ is a function defined on $M$ whose values are in $G^n$.”

p. 62, Problem 5.2.5. Change to $\nabla \cdot (xf(|x|)) = nf(|x|) + |x|f'(|x|)$.

p. 64, Problem 5.2.11b. Remove the word “both”.

p. 67, Exercise 5.14a. $\nabla \cdot f = \partial_1 f_1 + \partial_2 f_2$.

p. 69, Exercise 5.17b. Printing 1 only. Replace “The bases $\{w_r(r, \theta)\}$ and $\{w_\theta(r, \theta)\}$ are not in general orthogonal” with “In general, neither $\{w_r(r, \theta)\}$ nor $\{w_\theta(r, \theta)\}$ is an orthogonal basis.”

p. 70, Exercise 5.20. “Hint: For Part (a) use Exercises 5.17 and 5.18.”

p. 71, Figure 5.7. “$x(c_1, c_2, c_3)$” $\rightarrow$ “$x(c_1, c_2, u_3)$”

p. 71, Paragraph 4. In general: (i). Each basis vector $x^k$ is orthogonal to the surface formed by fixing the coordinate $u_k$. (ii). Each basis vector $x_j$ is tangent to the curve which is the intersection of the two surfaces formed by fixing in turn the coordinates other than $u_j$.

p. 72, formula for $\nabla f$ in cylindrical coordinates. “$r^{-1}\partial_r f_\theta$” $\rightarrow$ “$r^{-1}\partial_\theta f$”.

p. 74, Problem 5.4.2. Better: “Hint: If $B$ is a blade, then $B^{-1} = B/B^2$, where $B^2$ is a scalar.”
p. 74, Problem 5.4.5. Delete. Renumber Problems 5.4.6-5.4.8 to 5.4.5-5.4.7.

p. 75, line 3. “manifolds in $\mathbb{R}^m$” → “manifolds in $\mathbb{R}^n$”.

p. 76, Problem 5.5.1. “to the unit sphere” → “in $\mathbb{R}^2$”
“$D = P_T(\partial)$” → “$DF = P_T(\partial F)$”.

p. 81, Problem 6.1.4.
“$[\bar{m} \bar{b}]$” → “$[\bar{m} \bar{b}]$”.

p. 82, first paragraph “substitute the result in $x^2/8 + y^2/2$” → “substitute the result in $xy$”

p. 82, Theorem 6.7, proof.
“Let $x(\xi)$ parameterize . . . $x(t) = x(\xi(t))$ parameterize a curve” → “Let $x(t)$ be a parameterized curve”.

p. 83, Problem 6.2.3. “on the triangle” → “inside the triangle”.
Ans. Max 20, Min 4.

p. 89, last line of the displayed equation in the proof of Theorem 7.3
$|P| \to 0$ to $|P| \to 0$.

p. 89. Replace the bulleted points with

- A definite integral of $f$, a number. It is the limit of sums (Definition 7.1). The number can represent areas, masses, etc.
- An indefinite integral of $f$, a function. It is an $F$ such that $F' = f$.

p. 95, Definition 7.8. “tangent line to $S$” → “tangent line to $C$”.

p. 117, Problem 8.2.3. Should read $x^2/a^2 + y^2/b^2 + z^2/c^2 \leq 1$.

p. 119, first two paragraphs. $f \to F$ in the integrands.

p. 122, Corollary 9.4, statement. “$S \subset \mathbb{R}^m$” → “$S \subset \mathbb{R}^3$.”

p. 130, Exercise 10.2.
“defined on the boundary” → “defined on $M$ and $\partial M$”.

p. 133, toward bottom. “which is wanting in the definition given by Eq. (5.4).”

p. 134, Problem 10.2.4b. Change vector field $f$ to scalar field $f$. Drop Part c.

p. 136, Corollary 10.5, Proof. “Step (2) uses LAGA Theorem 6.30c and $dS^* = d\sigma$.”
→ “Step (2) uses LAGA Theorem 6.30c.”

p. 138, toward bottom.
“which is wanting in the definition given by Eq. (5.5).”

p. 141, top. “manifolds of arbitrary dimension.” → “$\mathbb{R}^m$.”

p. 142, Theorem 10.10, statement. Serious error!
“Let $M$ be an $m$-dimensional manifold.” → “Let $M$ be a bounded open set in $\mathbb{R}^m$.”

$$F(x_0) = \frac{(-1)^{m+1}}{\Omega_m I_m} \int_{\partial M} \frac{x - x_0}{|x - x_0|^m} d^{m-1}x F(x),$$

Corollary 10.11, statement.
“Let $M$ be a manifold.” → “Let $M$ be a bounded open set in $\mathbb{R}^m$.”
p. 148, Theorem 11.5, first sentence of statement. Change to “Let $x(s)$ and $\bar{x}(s)$, $0 \leq s \leq L$, parameterize curves $C$ and $\bar{C}$.”

p. 150, Theorem 11.8, proof. $\ell(C) = \int_{[a,b]} |x'(u_1(t), u_2(t))|\,dt$

$|x'(u_1, u_2)|^2 = x'(u_1, u_2) \cdot x'(u_1, u_2) = \ell(C)$

p. 152, Theorem 5.18, proof. $f'(h) \rightarrow f_p'(h)$.

p. 152, Exercise 11.31a. Show that the metric $G(r, \theta) = [r \ 0] [0 \ 1]$.

p. 161, Figure 11.6. Remove the hat on $p_1$ and $p_2$.

p. 163, Standard Terminology. “The metric $G$ (Eq. (11.5))” $\rightarrow$ “The expression $ds^2 = g_{ij}du_idu_j$ (Eq. (11.7))”.

p. 172, Differentiation entry. “print diff(diff(x**2, x), y)” $\rightarrow$ “print diff(diff(y*x**2, x), y)”.

p. 172, Jacobian entry. Redo: Jacobian. Let $X$ be an $m \times 1$ matrix of $m$ variables. Let $Y$ be an $n \times 1$ matrix of functions of the $m$ variables. These define a function $f: X \in \mathbb{R}^m \mapsto Y \in \mathbb{R}^n$. Then $Y.jacobian(X)$ is the $n \times m$ matrix of $f'_x$, the differential of $f$.

$r, \ theta = symbols('r \ theta')$
$X = Matrix([r, \ theta])$
$Y = Matrix([r*cos(\ theta), \ r*sin(\ theta)])$
$print Y.jacobian(X) \# Print $2 \times 2$ Jacobian matrix.$
$print Y.jacobian(X).det() \# Print Jacobian determinant (only if $m = n$).

Sometimes you want to differentiate $Y$ only with respect to some of the variables in $X$, for example when applying Eq. (3.24). Then include only those variables in $X$. For example, using $X = Matrix([r])$ in the example above produces the $2 \times 1$ matrix $[\cos \theta \ sin \theta]$.

p. 172, Iterated Integrals entry.

“make_symbols('x y')” $\rightarrow$ “x, y = symbols('x y')”.

p. 174, Reciprocal Basis entry. Drop everything after the first sentence.

p. 175, Change to Compute the vector derivative $\partial f$, divergence $\partial \cdot f$, curl $\partial \wedge f$: $M.grad * f$, $M.grad < f$, $M.grad \wedge f$. 