May 8, 2020

Errata for *Vector and Geometric Calculus*
Printings 1-5

p. 81. Replace the multivector $G$ with the vector $g$ and the multivector $\bar{G}$ with the vector $\bar{g}$ everywhere. And in the statement of Theorem 5.27, $\partial \cdot h \to h \cdot \partial$.

p. 155, Exercise 10.17. Replace with
Let $F(x, y) = u(x, y) + v(x, y)i$, where $u$ and $v$ are scalar valued and $i$ is the unit pseudoscalar of the $xy$-plane. Suppose that $F$ is analytic. Prove the Cauchy-Riemann equations: $u_x - v_y = 0$ and $v_x + u_y = 0$.

Errata for *Vector and Geometric Calculus*
Printings 1-4

Note: “p. m (n)” refers to page $m$ of Printing 4 and page $n$ of Printings 1-3.

p. 30, proof of Theorem 3.8. “the last sum in Eq (3.6) approaches 0.”
→ “the right side of Step (3), divided by $|h|$, approaches 0 with $h$.”

p. 31 (29), just before Theorem 3.10. $f'_x(h) = [f'_x][h] \to [f'_x(h)] = [f'_x][h]$.

p. 35 (33), proof of Theorem 3.14. (Problem 8.1.14) \to (LAGA Problem 8.1.14).

p. 44, bottom. “for $y(x)$ near $a$” \to “for $y$ in terms of $x$ near $a$”.

p. 56 (54), an omission, not an error. New second paragraph after Definition 4.5: “The tangent space is a vector space (LAGA Exercise 8.12).”

p. 58 (55), caption of Figure 5.14. “$f'$ maps” \to “$f'_p$ maps”.

p. 62 (58), Definition 5.2. Add a footnote after the first line: “i.e., the scalar coefficients of $F$ are differentiable.”

p. 64, Problem 5.2.10. Replace “$f'_x(a)$” with “$f'_p(a)$”.

p. 79 (73), Problem 5.5.1a. “Eq. (6.25)” \to “Eq. (5.19)”.

p. 85, first line of the proof of Theorem 6.5, improved: “First, $\partial_i f(x)h_i = \nabla f(x) \cdot h = 0$.”

p. 86, Definition 6.3. In “a strict local minimum at $x$ if $f(x) \leq f(x + h)$” change to “$f(x) < f(x + h)$”.

p. 96, Exercise 7.1. The approximate value of the integral is 0.2570120954.

p. 99, Problem 7.1.1. At end: “Parts (c)-(d)”.

p. 105. Remove Problem 7.2.9. It is a duplicate of Problem 7.2.3.

p. 115, Problem 7.2.3. Integral should read $\int_{[a, b]} \sqrt{r^2 + r^2} \, d\theta$.

p. 129, just before Figures 8.2-8.4. Strike “to a scalar”.


p. 131, Problem 9.1.4. Displayed equation should read $2\pi \int_a^b \sqrt{1 + r'(z)^2} r(z) \, dz$.

p. 132. The formula for area in the middle of the page should be $\iint_A |x_u(u,v) \wedge x_v(u,v)| \, dA$.

p. 138, Problem 9.2.2. The answer is $-4\pi/3$.

p. 142, last paragraph, first sentence. “multiple integrals” → “directed integrals”.

p. 144, Exercise 10.2. Replace the hint with “Fact: The boundary of a boundary is empty: $\partial(\partial M) = \emptyset.$”

p. 147, improved version of Corollary 10.3 (Generalized divergence theorem). Let $M$ be a bounded $m$-dimensional manifold in $\mathbb{R}^m$ and $\mathbf{n}$ be its outward normal. Let $F$ be a multivector field on $M$. Set $d\sigma = \mathbf{n} d^{m-1} x$. Then

$$\int_M \nabla \cdot F \, d^m x = \oint_{\partial M} d\sigma \cdot F.$$

**Proof.** Suppose first that $F$ is of a single grade, $g$. Use the extended fundamental identity (LAGA Theorem 6.28) to split the geometric product in both integrands of Eq. (10.5) into inner and outer products, of grades $g-1$ and $g+1$ respectively. Equating the inner product parts gives Eq. (10.7) for a single grade $F$.

To finish, apply this to each grade of a multigrade $F$ and add. \qed

p. 153, first displayed equation: $(\nabla \times f)(x^*) \cdot \mathbf{n} \rightarrow ((\nabla \times f) \cdot \mathbf{n})(x^*)$. And the left side of Eq. (10.14) should read: $((\nabla \times f) \cdot \mathbf{n})(x_0)$.

p. 166, Theorem 11.8. “Let $x(u(t), v(t))$” → “Let $x(u_1(t), u_2(t))$.”
Errata for *Vector and Geometric Calculus*
Printings 1-3

p. 30, Theorem 3.10, first line of proof:

\((\partial_i f_1(x), \ldots, \partial_i f_n(x)) \rightarrow (\partial_i f_1(x), \ldots, \partial_i f_m(x))\).

p. 40, just before the problems. "series of a vector valued function f centered" → "series of an f centered".

p. 44, first line. "differentiable" → "continuously differentiable"

p. 51, Theorem 4.3 should read:

Let \(x_1(t)\) and \(x_2(t)\) parameterize curves and suppose that \(x'_1(t)\) and \(x'_2(t)\) exist.

Then \((x_1(t)x_2(t))' \) exists and

\[
(x_1(t)x_2(t))' = x'_1(t)x_2(t) + x_1(t)x'_2(t).
\]

Change Eq. (4.6) similarly. Remove the paragraph following the theorem.

p. 55, change bottom to

Let \(f\) be a 1-1 map between manifolds of equal dimension.

Then the tangent map \(f'_p\) maps \(T_p\) to \(T_{f(p)}\).

p. 58. Remove Problem 4.3.6 Part (b) and restate Part (a):

The definition of the determinant of a linear transformation on a vector space (LAGA Definition 8.21) does not apply to the linear transformation \(f'_p\) because it is between two vector spaces. Define \(\det(f'_p)\). Hint: See LAGA Definition 8.21.

p. 64, Problem 5.2.11. \(\nabla \wedge e = -\partial_t B \rightarrow \nabla \wedge e = \partial_t B\).

p. 76, Problem 11.2.7. New Part (b): Define a directional derivative for fields defined on a surface by \(\partial_h f(p) = (h \cdot \partial)f(p)\) (Definition 11.10). Compute \(\partial_t t\) on the equator. Ans. \(-\sin \theta i + \cos \theta j)/\rho\).

Note that at the equator \(t\) is in the tangent plane but \(\partial_t t\) is not.

p. 82, Theorem 6.7 statement. \(g(x_0) = c \rightarrow g(x) = c\).

p. 105, Problem 5.4.1. Field should be \(e^x(\sin(xy) + y \cos(xy))i + xe^x \cos(xy)j\)

p. 116, Exercise 8.8. The answer is \(\pi(e^4 - 1)\).

p. 132, Corollary 10.3. "Let \(f\) be a multivector field" → "Let \(f\) be a vector field"

p. 134, Problem 10.2.6c. \(\oint_C e \cdot ds = \partial_t \iint_S B \cdot dS \rightarrow \oint_C e \cdot ds = -\partial_t \iint_S B \cdot dS\)

p. 186. Add "65" and "66" to gradient entry. Add "98" to curl entry. Add "124" to divergence entry.
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Printings 1-2

p. 14, line 8: Delete "in".

p. 23, line 4: Change the period after “tangents” into a comma.

p. 28, Exercise 3.9: Change “Eq. (3.23)” to “Eq. (3.18)”. 

p. 30, Exercise 3.13. \( f : U \subseteq \mathbb{R}^n \rightarrow \mathbb{R}^m \rightarrow f : \mathbb{R}^n \rightarrow \mathbb{R}^m \). 

p. 38, Problem 3.4.12c. Change to “The ideal gas law is \( pv = nRT \), where \( n, p, v, T \) are the number of moles, pressure . . . .” 

p. 43, last two lines. \( f(a, b) = 0 \rightarrow f(a, b) = (2, 2) \). 

p. 49, Definition 4.1. “Let \( q \in A \) and set \( p = x(q) \)” → “Let \( t \in A \) and set \( p = x(t) \).” 

p. 55, Figure 5.14, caption. “onto \( S \)” → “to \( S \).” 

p. 59, Corollary 5.7. \( f^* (b) \rightarrow f^* x(b) \). 

p. 59, line -5: Change “Eq. (5.2) and Eq. (3.6)” to “Eq. (5.2) and Eq. (3.23)”. 

p. 64, bottom. Problem 4.3.12 → LAGA Problem 4.3.12 

p. 69, Eq. (5.17). \( \partial x_i^{\ell} \partial w_j \epsilon_i \rightarrow \partial x_\ell \partial w_j \epsilon_i \). 

p. 72, Exercise 5.36. Equations should read \( \hat{\phi} = \cos \phi (\cos \theta i + \sin \theta j) - \sin \phi k, \quad \hat{\theta} = - \sin \theta i + \cos \theta j. \) 

p. 75, footnote. \( \mathbb{R}^3 \rightarrow \mathbb{R}^n \). 

p. 76, Problem 5.6.1b, second printing only. “even though \( \text{hat} t \) is” → “even though \( \hat{t} \) is”. 

p. 79, Theorem 6.5. A much better proof:  
**Proof.** Since \( \nabla f(x) = 0, \partial_i f(x)h_i = 0 \). And \( \partial_{ij} f(x)h_i h_j > 0 \) for \( h \neq 0 \), since \( Hf(x) \) is positive definite. Then \( \partial_{ij} f(x + t^*h)h_i h_j > 0 \) for small \( t^*h \neq 0 \), since the partial derivatives are continuous at \( x \). The theorem now follows from Eq. 3.2. 

p. 81, Problem 6.1.2c. \( \lim_{(x,y) \rightarrow \infty} \rightarrow \lim_{(x,y) \rightarrow \infty} f(x, y). \) 

p. 90, equation mid-page, \( \int_{[a,b]} a f dx = a \int_{[a,b]} f dx \rightarrow \int_{[a,b]} cf dx = c \int_{[a,b]} f dx. \) 

p. 91, after the sentence beginning with Think of. “Divide \( C \) into infinitesimal parts. Multiply the value of \( F \) on each part by the infinitesimal length \( ds \) of the part. Add to form the integral.”

p. 96, note at the bottom of the page. “here here” → “here”. 

p. 98, Problem ??c. Misplaced “): “\( F(x(u,v)x_u(u,v)) \)” → “\( F(x(u,v))x_u(u,v) \).”

p. 100, Exercise ??c. “Theorem 7.11” → “Theorem 7.10”.

p. 100, proof of Theorem 7.13: “it independent” → “it is independent”. 
p. 101, Exercise 5.20b. “not conservative” → “not conservative in \( \mathbb{R}^2 - \{0\} \)

p. 102, following Definition 5.18. “All simple closed curves” → “All closed curves”.

p. 103, line -7. Switch “m” and “M”.

p. 103, line -5: Change “Eq. (7.10)” to “(Eq. (7.14))”.

p. 107, line -3: Change “set open” to “open set”.

p. 112, Exercise 8.2. \( \int_{y=0}^{1} f(y) \) → \( \int_{y=0}^{2} f(y) \).

p. 124, Problem 9.2.3: “scalar + trivector” → “vector + trivector”.

p. 131, Fig. 10.6: Arrows should be reversed, as the \( M_i \) are “oriented clockwise”.

p. 136, second line of the proof of Corollary 10.5: \((-1)^{2\times2} \) → \((-1)^{2\times1}\).

p. 142, below Corollary 10.10. \( f(x) = \int_{a}^{x} f'(t) dt + f(a) \).

p. 146, lines 12 and 17: Change “Theorem 4.3b” and “Theorem 4.3” (both) to “Eq. (4.6)”.

p. 148, Theorem 11.5, Proof. \( \ddot{x} \) → \( \dot{x} \), twice

p. 151, line above Def 11.10: Change “Definition 5.23” to “Definition 5.15”.

p. 158, middle displayed line in the proof of Theorem 11.25. Drop the middle term.


p. 162, line 5. Delete “is”.
Errata for *Vector and Geometric Calculus*

Printing 1

Due to publisher error the shading in several of figures is washed out in some copies of the book. The correct shading is shown below. I think that if Figure 2.1 is OK in your book, then all figures are.

p. 4, Exercise 1.1. “$c_{ij} \rightarrow c_{ik}$”.

p. 6, Problem 1.1.1. “$x(t) \rightarrow x(\theta)$”.
p. 8, Exercise 1.11. Delete "We will do this often." Add "This will allow us to specialize formulas for surfaces defined parametrically to surfaces defined by z = f(x, y). Exercise 5.38 is an example."

p. 11, Exercise 1.14. "x ≠ 0" → "x > 0".

φ = arccos(z/r) → φ = arctan(r/z).

p. 22, bottom. Remove "The definition shows that ∂iF has the same grades as F." Parts disappear if their partial derivative is zero.

p. 23, Exercise 3.1. "REm to Rn" → "Rm to Rn".

p. 33, Problem 3.2.1. Append the sentence "Then for fixed x, the differential is the linear transformation h ↦→ f′(x)h."

p. 33, Problem 3.2.3. (ρ, θ, φ) → (ρ, φ, θ).

p. 34, first and second displayed equations should read
x, h ∈ Rn ⇒ (g ◦ f)(x) ∈ Rp ⇒ (g ◦ f)′(x)(h) ∈ Rp,
x, h ∈ Rn ⇒ f′x(h) ∈ Rm ⇒ (g′f(x) ◦ f′x)(h) ∈ Rp.

p. 34. Line should read \[ \frac{4}{\ldots} + [g′(R(h)) |h| + S(k_h)|k_h|]. \]

p. 34. Replace the end of the page with the following:
The added phrase "divided by |h|" is the reason for the changes. To finish, we show that the term in brackets above, divided by |h|, approaches zero with |h|. First, using the continuity of g' (Theorem 2.10),
\[ \lim_{h \to 0} g′(R(h)) = g′( \lim_{h \to 0} R(h)) = g′(0) = 0. \]
Second, with |f′| being the operator norm of f',
\[ |k_h| \leq |f′(h)| + |R(h)||h| \leq |f′| |h| + |R(h)| |h|. \]
Thus, since (h → 0) ⇒ (k_h → 0) ⇒ (S(k_h) → 0), \[ \lim_{h \to 0} |S(k_h)||k_h|/|h| = 0. \]

p. 37, statement of Theorem 3.16. "Then the inverse function (f′a)−1n → "Then the inverse function f−1n."

p. 40, following Theorem 3.13: "In other words, ∂h f is linear in both h and f."

p. 40, Problem 3.3.5. The variable names I used lead to confusion. Change to f(x, y) = (x cos y, x sin y). And add "(All coordinates are cartesian.)"

p. 40, Problem 3.3.3. "continuity of f at x." → "continuity of f at x."

p. 41, Statement of Theorem 3.19. "has a differentiable inverse" → "has a continuously differentiable inverse".

p. 42, third line. Remove "there is a neighborhood of each y_i in which".

p. 45, Exercise 3.6.1b. "Determine ∂ρ/∂x."

p. 48, below Eq. (4.3). "of higher dimension" → "in higher dimensions".
pp. 49-55. Equation (4.4) in Section 4.1 established the notation of m-dimensional manifolds \( M \) as subsets of \( \mathbb{R}^n \). However, Sections 4.2 and 4.3, while mostly internally consistent, are inconsistent with this notation. The following changes remove the inconsistency:

- p. 49, 2nd paragraph: "a curve \( C \) in \( \mathbb{R}^n \)."
- p. 50, Theorem 4.2: \( m \) to \( n \).
- p. 51, Theorem 4.4: \( \mathbb{R}^m \) to \( \mathbb{R}^n \).
- p. 52, Theorem: \( \mathbb{R}^m \) to \( \mathbb{R}^n \), \( \mathbb{R}^n \) to \( \mathbb{R}^n \).
- p. 53, First sentence and left part of Figure 4.5: \( \mathbb{R}^m \) to \( \mathbb{R}^n \).
- p. 53, Definition 4.5: \( \mathbb{R}^m \) to \( \mathbb{R}^n \).
- p. 53, line following Eq. (4.11): \( m \) to \( n \).
- p. 55, Theorem: \( \mathbb{R}^m \) to \( \mathbb{R}^n \), \( \mathbb{R}^n \) to \( \overline{\mathbb{R}}^n \).

p. 49, Paragraph 2. "Let \( x(t) \) parameterize a curve \( C \). Then \( x \) has a nonzero differential (p. 49). By Problem 3.2.1 it is . . . ”

p. 50, line after Theorem 4.3. “Definition 4.5” → “Eq. (4.5)”

p. 52, Theorem. “\( f' \) is one-to-one.” → “\( f' \) restricted to \( T_p \) is one-to-one.”

p. 52, Problem 4.2.3.
- a. Show that components of \( \Omega \) must have grade 2 or 3.
- b. Show that components of \( \Omega \) must have grade 0 or 2.

p. 53, Eq. (4.10). “\( \lim_{h \to 0} \)” → “\( \lim_{h \to 0} \)”

p. 53, sentence below Definition 4.5. “Recall that the differential \( x'_q \) is one-to-one and maps linearly independent vectors . . . ”

p. 55, Theorem. “\( f' \) is one-to-one.” → “\( f' \) restricted to \( T_p \) is one-to-one.”

p. 56, Problem 4.3.1b. \( x_u \land x_v \to x_\phi \land x_\theta \).

p. 57, Definition 5.1. “Let \( M \) be a manifold in \( \mathbb{R}^n \). A field on \( M \) is a function defined on \( M \) whose values are in \( G^n \).”

p. 62, Problem 5.2.5. Change to \( \nabla \cdot (xf(|x|)) = nf(|x|) + |x|f'(|x|) \).

p. 64, Problem 5.2.11b. Remove the word “both”.

p. 67, Exercise 5.14a. \( \nabla \cdot f = \partial_1 f_1 + \partial_2 f_2 \).

p. 69, Exercise 5.17b. Printing 1 only. Replace “The bases \( \{w_r(r, \theta)\} \) and \( \{w_\theta(r, \theta)\} \) are not in general orthogonal” with “In general, neither \( \{w_r(r, \theta)\} \) nor \( \{w_\theta(r, \theta)\} \) is an orthogonal basis.”

p. 70, Exercise 5.33. “Hint: For Part (a) use Exercises 5.30 and 5.31.”

p. 71, Figure 5.11. “\( x(c_1, c_2, c_3) \)” → “\( x(c_1, c_2, u_3) \)”

p. 71, Paragraph 4. In general: (i). Each basis vector \( x^k \) is orthogonal to the surface formed by fixing the coordinate \( u_k \). (ii). Each basis vector \( x_j \) is tangent to the curve which is the intersection of the two surfaces formed by fixing in turn the coordinates other than \( u_j \).

p. 72, formula for \( \nabla f \) in cylindrical coordinates. “\( r^{-1} \partial_{\theta} f \)” → “\( r^{-1} \partial_{\theta} f \)”

p. 74, Problem 5.5.2. Better: Hint: If \( B \) is a blade, then \( B^{-1} = B/B^2 \), where \( B^2 \) is a scalar.
p. 74, Problem 5.4.5. Delete. Renumber Problems 5.4.6-5.4.8 to 5.4.5-5.4.7.

p. 75, line 3. “manifolds in $\mathbb{R}^m$” → “manifolds in $\mathbb{R}^n$”.

p. 76, Problem 5.6.1. “to the unit sphere” → “in $\mathbb{R}^2$”
“$D = P_T(\partial)$” → “$DF = P_T(\partial F)$”.

p. 81, Problem 6.1.4. $[m_b]$ → $[\bar{m} \bar{b}]$.

p. 82, first paragraph “substitute the result in $x^2/8 + y^2/2$” → “substitute the result in $xy$”

p. 82, Theorem 6.7, proof.
“Let $x(\xi)$ parameterize . . . $x(t) = x(\xi(t))$ parameterize a curve ” → “Let $x(t)$ be a parameterized curve”.

p. 83, Problem 6.2.3. “on the triangle” → “inside the triangle”.

p. 89, last line of the displayed equation in the proof of Theorem 7.3 $|P| \to^0$ to $|P| \to^0$.

p. 95, Definition 7.8. “tangent line to $S$” → “tangent line to $C$”.

p. 117, Problem 8.2.3. Should read $x^2/a^2 + y^2/b^2 + z^2/c^2 \leq 1$.

p. 119, first two paragraphs. $f \to F$ in the integrands.

p. 122, Corollary 9.4, statement. “$S \subset \mathbb{R}^m$” → “$S \subset \mathbb{R}^3$.”

p. 130, Exercise 10.2.
“defined on the boundary” → “defined on $M$ and $\partial M$”.

p. 133, toward bottom. “which is wanting in the definition given by Eq. (5.3).”

p. 134, Problem 10.2.4b. Change vector field $f$ to scalar field $f$. Drop Part c.

p. 136, Corollary 10.5, Proof. “Step (2) uses LAGA Theorem 6.30c and $dS^* = d\sigma$.” → “Step (2) uses LAGA Theorem 6.30c.”

p. 138, toward bottom.
“which is wanting in the definition given by Eq. (5.3).”

p. 141, top. “manifolds of arbitrary dimension.” → “$\mathbb{R}^m$.”

p. 142, Theorem 10.10, statement. Serious error!
“Let $M$ be an $m$-dimensional manifold.” → “Let $M$ be a bounded open set in $\mathbb{R}^m$.”

$$F(x_0) = \frac{(-1)^{m+1}}{\Omega_m I_m} \oint_{\partial M} \frac{x - x_0}{|x - x_0|^m} d^{m-1}x F(x),$$

Corollary 10.11, statement.
“Let $M$ be a manifold.” → “Let $M$ be a bounded open set in $\mathbb{R}^m$.”
p. 148, Theorem 11.5, first sentence of statement. Change to "Let \( x(s) \) and \( \bar{x}(s) \), \( 0 \leq s \leq L \), parameterize curves \( C \) and \( \bar{C} \)."

p. 150, Theorem 11.8, proof. \[ \ell(C) = \int_{[a,b]} |x'(u_1(t), u_2(t))| \, dt \]
\[ |x'(u_1, u_2)|^2 = x'(u_1, u_2) \cdot x'(u_1, u_2) = \]

p. 152, Theorem 5.25, proof. \[ f'(h) \to f'_p(h) \]

p. 152, Exercise 11.24a. Show that the metric \( G(r, \theta) = [r \ 0 \ 0 \ 1] \).

p. 161, Figure 11.3. Remove the hat on \( p_1 \) and \( p_2 \).

p. 163, Standard Terminology. "The metric \( G \) (Eq. (11.5))" → "The expression \( ds^2 = g_{ij} du_i du_j \) (Eq. (11.7))".

p. 172, Differentiation entry. "print diff(diff(x**2,x),y)" → "print diff(diff(y*x**2,x),y)".

p. 172, Jacobian entry. Redo:

\textbf{Jacobian.} Let \( X \) be an \( m \times 1 \) matrix of \( m \) variables. Let \( Y \) be an \( n \times 1 \) matrix of functions of the \( m \) variables. These define a function \( f: X \in \mathbb{R}^m \mapsto Y \in \mathbb{R}^n \). Then \( Y.jacobian(X) \) is the \( n \times m \) matrix of \( f'_x \), the differential of \( f \).

\begin{verbatim}
r, theta = symbols('r theta') X = Matrix([r, theta]) Y = Matrix([r*cos(theta), r*sin(theta)])
print Y.jacobian(X)  # Print 2 \times 2 Jacobian matrix.
print Y.jacobian(X).det()  # Print Jacobian determinant (only if \( m = n \)).
\end{verbatim}

Sometimes you want to differentiate \( Y \) only with respect to some of the variables in \( X \), for example when applying Eq. (3.24). Then include only those variables in \( X \). For example, using \( X = \text{Matrix}([r]) \) in the example above produces the \( 2 \times 1 \) matrix \( \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix} \).

p. 172, Iterated Integrals entry. "make_symbols('x y')" → "x, y = symbols('x y')".

p. 174, Reciprocal Basis entry. Drop everything after the first sentence.

p. 175. Change to Compute the vector derivative \( \nabla f \), divergence \( \nabla \cdot f \), curl \( \nabla \wedge f \):
\( \text{M.grad} \ast f \), \( \text{M.grad} \cdot f \), \( \text{M.grad} \wedge f \).