Math 141 Exam One Spring 2014

Solutions

1. $\tan x = -2 \Rightarrow \dfrac{\sin x}{\cos x} = -2 \Rightarrow \sin x = -2\cos x$

$\sin^2 x + \cos^2 x = 1 \Rightarrow (-2\cos x)^2 + \cos^2 x = 1$

$\Rightarrow 4\cos^2 x + \cos^2 x = 1 \Rightarrow 5\cos^2 x = 1 \Rightarrow \boxed{\cos x = \dfrac{1}{\sqrt{5}}}$

because x is in Q4. $\boxed{\sin x = \dfrac{-2}{\sqrt{5}}}$

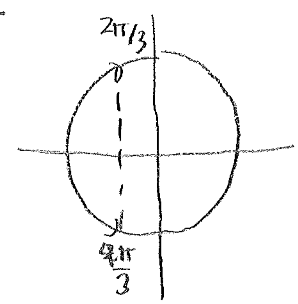
2. $y - y_0 = m(x - x_0)$. $x_0 = \dfrac{\pi}{2}$, $y_0 = \cos\dfrac{\pi}{2} + \dfrac{\pi}{2} = \dfrac{\pi}{2}$

$m = f'(x_0) = -\sin\dfrac{\pi}{2} + 1 = -1 + 1 = 0$

$\therefore\ y - \dfrac{\pi}{2} = 0\left(x - \dfrac{\pi}{2}\right) \Rightarrow \boxed{y = \dfrac{\pi}{2}}$

3. Solve $\cos x + \frac{1}{2} < 0 \Rightarrow \cos x < -\frac{1}{2}$

$\boxed{\dfrac{2\pi}{3} < x < \dfrac{4\pi}{3}}$



4. $\cos 2\theta + \cos\theta = 0 \Rightarrow \cos^2\theta - \sin^2\theta + \cos\theta = 0$

$\Rightarrow \cos^2\theta - (1 - \cos^2\theta) + \cos\theta = 0 \Rightarrow 2\cos^2\theta + \cos\theta - 1 = 0$

$\Rightarrow (2\cos\theta - 1)(\cos\theta + 1) = 0 \Rightarrow \cos\theta = \frac{1}{2}$ or $\cos\theta = -1$

$\boxed{\theta = \dfrac{\pi}{3} + 2n\pi,\ \dfrac{5\pi}{3} + 2n\pi,\ \theta = \pi + 2n\pi}$

5. $\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos\frac{\pi}{4}$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

6. (a) $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A\cos B + \sin B\cos A}{\cos A\cos B - \sin A\sin B}$ divide by $\cos A\cos B$

$= \frac{\frac{\sin A\cancel{\cos B}}{\cos A\cancel{\cos B}} + \frac{\sin B\cancel{\cos A}}{\cancel{\cos A}\cos B}}{1 - \frac{\sin A\sin B}{\cos A\cos B}} = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

(b) $\lim_{x\to\frac{2\pi}{3}} \frac{\sin x - \sqrt{3}/2}{x - \frac{2\pi}{3}} = \lim_{x\to\frac{2\pi}{3}} \frac{\sin x - \sin\frac{2\pi}{3}}{x - \frac{2\pi}{3}} = \frac{d}{dx}\sin x\Big|_{x=\frac{2\pi}{3}} = \cos\left(\frac{2\pi}{3}\right) = \boxed{-\frac{1}{2}}$

7. (a) $f'(x) = \boxed{\cos x - x\sin x + \sec^2 x}$

(b) $g(\theta) = 2\theta^{-\frac{1}{2}} + \sin\theta$, $g'(\theta) = -\theta^{-3/2} + \cos\theta$

$g''(\theta) = \boxed{\frac{3}{2}\theta^{-5/2} - \sin\theta}$

(c) $f'(t) = \boxed{\frac{-\sin t(1-\sin t) - \cos t(-\cos t)}{(1-\sin t)^2}}$

(d) $f'(x) = \boxed{\frac{1}{2\sqrt{x^2+\sin x+1}} \cdot (2x + \cos x)}$

(e) $\frac{dp}{dq} = \boxed{\frac{(\sin q + q\cos q)(q^2-1) - q\sin q(2q)}{(q^2-1)^2}}$

8. (a) amplitude $= \frac{1}{2}$, period $= \frac{2\pi}{2} = \pi$, horizontal shift $\frac{\pi}{2}$ right

vertical shift $= 0$

8. Let $f(x) = \frac{1}{2}\sin(2x - \pi)$.

(a) Specify the amplitude, period, horizontal shift, and vertical shift of the graph of f. $\boxed{\text{5 points}}$

(b) Sketch the graph of f on the grid below for the interval $-2\pi \le x \le 2\pi$. Be sure the indicate the vertical **scale** on the grid. $\boxed{\text{10 points}}$

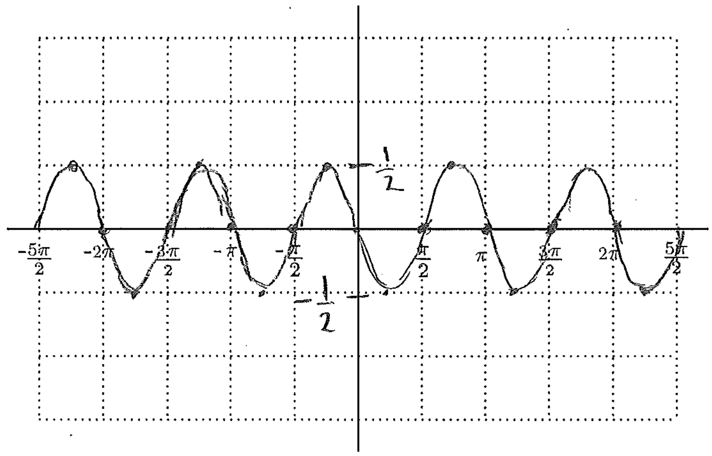


Figure 1: Grid for problem 8.