Math 141 A

Exam 2
Solutions

Spring 2014

1. (a) $x^2 + y = 4xy \Rightarrow 2x + y' = 4y + 4xy' \Rightarrow 4xy' - y' = 2x - 4y$

$\Rightarrow y'(4x-1) = 2x - 4y \Rightarrow \boxed{y' = \dfrac{2x-4y}{4x-1}}$

(b) $\ln\sqrt{y} = e^x + \arctan x \Rightarrow \dfrac{1}{\sqrt{y}} \cdot \dfrac{1}{2\sqrt{y}} y' = e^x + \dfrac{1}{x^2+1}$

$\Rightarrow \dfrac{1}{2y} y' = e^x + \dfrac{1}{x^2+1} \Rightarrow \boxed{y' = 2ye^x + \dfrac{2y}{x^2+1}}$

2. $y^2 - 2x - 4y - 1 = 0 \quad (x_0, y_0) = (-2, 1).$

Find $y'$: $2yy' - 2 - 4y' = 0 \Rightarrow (2y-4)y' = 2 \Rightarrow y' = \dfrac{2}{2y-4}$

$y'(-2,1) = \dfrac{2}{2(1)-4} = \dfrac{2}{-2} = -1.$

tangent line: $y - 1 = -(x+2)$

normal line: $y - 1 = (x+2)$

$m_t = -1$

$m_n = \dfrac{-1}{m_t} = \dfrac{-1}{-1} = 1$

3. $r^2 + s + v^3 = 2 \Rightarrow 2r\dfrac{dr}{dt} + \dfrac{ds}{dt} + 3v^2\dfrac{dv}{dt} = 0$. Need

a value for $v$ when $r = 2$ and $s = -3$.

$2^2 + -3 + v^3 = 2 \Rightarrow 4 - 3 + v^3 = 2 \Rightarrow 1 + v^3 = 2 \Rightarrow v^3 = 1$

so $v = 1$. Now, $2(2)(2) + -3 + 3(1)^2\dfrac{dv}{dt} = 0$

$\Rightarrow 8 - 3 + 3\dfrac{dv}{dt} = 0 \Rightarrow 3\dfrac{dv}{dt} = -5 \quad \boxed{\dfrac{dv}{dt} = -\dfrac{5}{3}}$

4. (a) $x^2 + y^2 = 13^2$

$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = 0$

$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt} = \frac{-5}{\cancel{12}_6}(\cancel{2})$

$= \boxed{\frac{-5}{6}\ \text{ft/sec}}$

$\frac{dy}{dt} = ?$

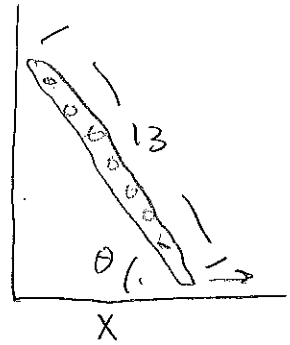
when $x = 5$

$y = \sqrt{13^2 - 5^2}$

$= \sqrt{169 - 25}$

$= \sqrt{144}$

$= 12$




(b) $\sin\theta = \frac{y}{13} \Rightarrow \theta = \arcsin\frac{y}{13} \Rightarrow \frac{d\theta}{dt} = \frac{1}{\sqrt{1 - \frac{y^2}{13^2}}}\left(\frac{1}{13}\right)\frac{dy}{dt}$

$\Rightarrow \frac{d\theta}{dt} = \frac{1}{\sqrt{\frac{13^2}{13^2} - \frac{12^2}{13^2}}}\left(\frac{1}{13}\right)\left(\frac{-5}{6}\right) = \frac{\cancel{13}}{\sqrt{13^2 - 12^2}}\frac{1}{\cancel{13}}\frac{5}{6} = \boxed{\frac{-1}{6}\ \text{rad/sec}}$

5. $f(t) = 12t - t^3 \Rightarrow f'(t) = 12 - 3t^2$

$f'(t) = 0 \Rightarrow 12 - 3t^2 = 0 \Rightarrow t^2 = 4$

$\Rightarrow t = \pm 2$

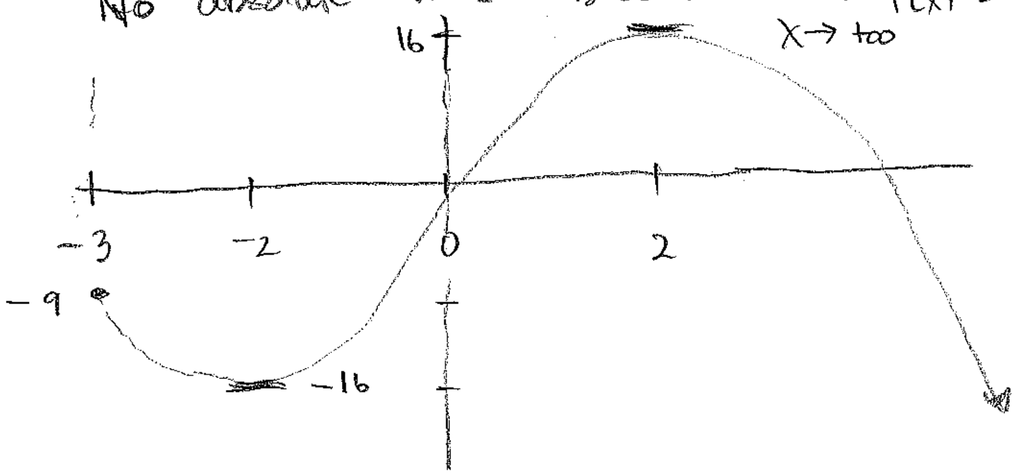
| | −3 | | −2 | | 0 | | 2 | |
|---|---|---|---|---|---|---|---|---|
| $f'$ | | − | 0 | + | | | 0 | − |
| | endpt max | | local min | | | | local max | |

(a)

| location | value | type |
|---|---|---|
| −3 | −9 | max |
| −2 | −16 | min |
| 2 | 16 | max |

(b) $\boxed{\text{Absolute max at } x = 2}$ based on the sign of $f'$ and the fact $16 > -9$.

No absolute min because $\lim_{x \to +\infty} f(x) = -\infty$



#6 $f''(x) = \sqrt{3} - 2\cos x = 0 \Rightarrow 2\cos x = \sqrt{3} \Rightarrow \cos x = \frac{\sqrt{3}}{2}$

$\Rightarrow x = \frac{\pi}{6}$ or $\frac{11\pi}{6}$ for $0 \le x \le 2\pi$

$f''$   $-$   0   $+$   0   $-$

$0 \quad \frac{\pi}{6} \quad \pi \quad \frac{11\pi}{6} \quad 2\pi$

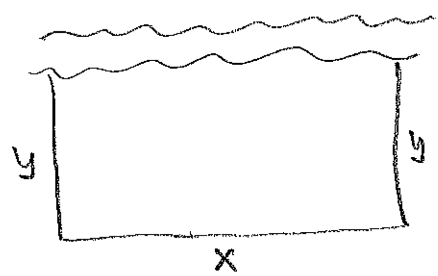
(a) $f$ CC↑ on $\left(\frac{\pi}{6}, \frac{11\pi}{6}\right)$

(b) $f$ CC↓ on $\left(0, \frac{\pi}{6}\right)$ and $\left(\frac{11\pi}{6}, 2\pi\right)$

(c) IPs: @ $x = \frac{\pi}{6}$ and $\frac{11\pi}{6}$

#7. $A$ = area

$A = xy$ maximiz




Constraint: $x + 2y = 1000$

$\Rightarrow x = 1000 - 2y$

$A(y) = y(1000 - 2y) = 1000y - 2y^2 \qquad 0 \le y \le 500$

$A'(y) = 1000 - 4y$

$A'(y) = 0 \Rightarrow 4y = 1000 \Rightarrow y = \frac{1000}{4} = 250$

$A'$   $+$   0   $-$

$0 \quad 250 \quad 500$

↑ abs max

Area maximized with

$$\boxed{\begin{aligned} y &= 250\text{ m} \\ x &= 1000 - 2(250) = 500\text{ m} \end{aligned}}$$