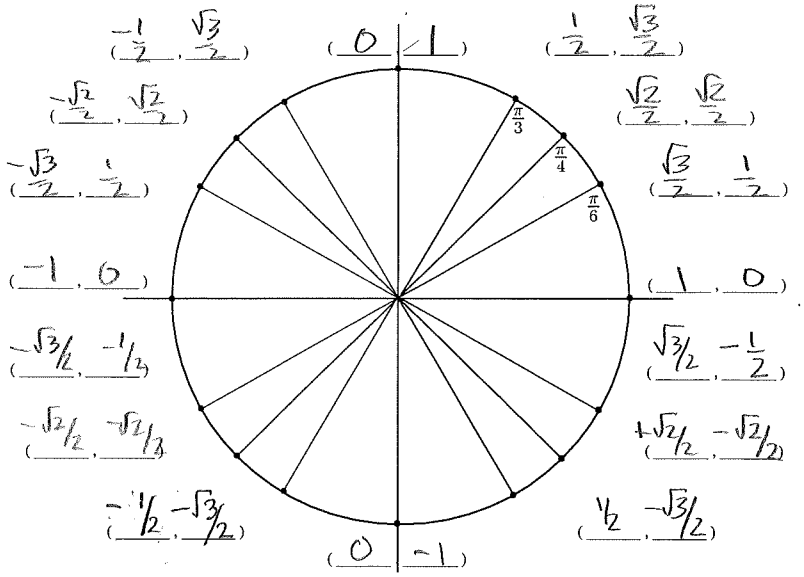
Name Solutions

Math 141A	Spring 2014

Quiz Three
February 21, 2014

DIRECTIONS:** Allow yourself no more than 30 minutes to complete this quiz. **No calculators.** This quiz is given under conditions of the ***Luther College Honor Code. You are expected to uphold the highest standards of academic integrity, and you are expected to demand the same from fellow students.

1. On the figure below, fill in the values of $\cos\theta$ and $\sin\theta$ in the appropriate blanks for the given angles. [10 points]



2. Calculate exact values for the following: [10 points]

(a) $\sin\left(\frac{11\pi}{12}\right)$

$$\frac{11\pi}{12} = \frac{8\pi}{12} + \frac{3\pi}{12} = \frac{2\pi}{3} + \frac{\pi}{4}$$

$$\sin\left(\frac{11\pi}{12}\right) = \sin\left(\frac{2\pi}{3} + \frac{\pi}{4}\right) = \sin\left(\frac{2\pi}{3}\right)\cos\frac{\pi}{4} + \cos\left(\frac{2\pi}{3}\right)\sin\frac{\pi}{4}$$

$$= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + \frac{1}{2}\frac{\sqrt{2}}{2}$$

$$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

(b) $\cos\left(\frac{-5\pi}{12}\right) = \cos\left(\frac{5\pi}{12}\right)$

$$\frac{5\pi}{12} = \frac{2\pi}{12} + \frac{3\pi}{12} = \frac{\pi}{6} + \frac{\pi}{4}$$

$$\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \cos\frac{\pi}{6}\cos\frac{\pi}{4} - \sin\frac{\pi}{6}\sin\frac{\pi}{4}$$

$$= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2}$$

$$= \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

3. Solve the following trigonometric equations to find all solutions in the interval $[0, 2\pi]$. **15 points**

(a) $2\cos^2 x - 1 = 0$ $\Rightarrow$ $2\cos^2 x = 1 \Rightarrow \cos^2 x = \frac{1}{2}$

$\Rightarrow \cos x = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$\cos x = \pm\frac{\sqrt{2}}{2} \Rightarrow$ $\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

(b) $\sin 2\theta + \cos\theta = 0$ $\Rightarrow$ $2\sin\theta\cos\theta + \cos\theta = 0$

$(2\sin\theta + 1)\cos\theta = 0$

$\Rightarrow \sin\theta = -\frac{1}{2}$ or $\cos\theta = 0$

$\boxed{\theta = \frac{7\pi}{6}, \frac{11\pi}{6} \quad \text{or} \quad \theta = \frac{\pi}{2}, \frac{3\pi}{2}}$

(c) $2\cos^2 x - \cos x - 1 = 0$ $\Rightarrow$ $(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -\frac{1}{2}$ or $\cos x = 1$

$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3} \quad \text{or} \quad x = 0, 2\pi}$

4. Find x such that $\arctan x + \sqrt{3} = 0$ $\Rightarrow$ $\arctan x = -\sqrt{3}$ **5 points**

$\tan(\arctan x) = \tan(-\sqrt{3})$

$x = \tan(-\sqrt{3})$

$= \boxed{-\tan\sqrt{3}}$