

Quiz Four
March 7, 2014

***DIRECTIONS:** Allow yourself no more than 30 minutes to complete this quiz. **No calculators.** This quiz is given under conditions of the *Luther College Honor Code*. You are expected to uphold the highest standards of academic integrity, and you are expected to demand the same from fellow students.

1. Suppose the functions f and g and their derivatives with respect to x have the following values at $x = 2$ and $x = 3$. 10 points

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	8	2	$1/3$	-3
3	3	-4	2π	5

Find the derivatives with respect to x of the following combinations at the given value of x .

- (a) $f(g(x))$, $x = 2$

$$\begin{aligned} \frac{d}{dx} f(g(x)) &= f'(g(x)) \cdot g'(x) \\ @ x=2 &= f'(g(2)) \cdot g'(2) \\ &= f'(2) \cdot -3 \\ &= (1/3)(-3) = \boxed{-1} \end{aligned}$$

- (b) $x\sqrt{f(x)}$, $x = 3$

$$\begin{aligned} \frac{d}{dx} x\sqrt{f(x)} &= \sqrt{f(x)} + x \cdot \frac{1}{2\sqrt{f(x)}} \cdot f'(x) \\ @ x=3 &= \sqrt{f(3)} + \frac{3}{2\sqrt{f(3)}} \cdot f'(3) = \sqrt{3} + \frac{3}{2\sqrt{3}} \cdot 2\pi \\ &= \boxed{\sqrt{3} + \sqrt{3}\pi} \end{aligned}$$

2. Find $\frac{dy}{dt}$ for $y = \sqrt{t + \cos(t^2)}$. 10 points

$$\begin{aligned} \frac{dy}{dt} &= \frac{1}{2\sqrt{t + \cos t^2}} \cdot (t + \cos t^2)' = \frac{1}{2\sqrt{t + \cos t^2}} (1 + (\cos t^2)') \\ &= \boxed{\frac{1}{2\sqrt{t + \cos t^2}} (1 - \sin t^2 \cdot 2t)} \end{aligned}$$

3. Use implicit differentiation to find an expression for dy/dx .

10 points

(a) $2xy + y^2 = x + y$

$$2y + 2xy' + 2yy' = 1 + y'$$

$$(2x + 2y - 1)y' = 1 - 2y$$

$$y' = \frac{1 - 2y}{2x + 2y - 1}$$

(b) $e^{2x} = \sin(x + 3y)$

$$e^{2x} \cdot 2 = \cos(x + 3y) (x + 3y)'$$
$$= \cos(x + 3y) (1 + 3y')$$

$$1 + 3y' = \frac{2e^{2x}}{\cos(x + 3y)}$$

$$y' = \frac{1}{3} \left[\frac{2e^{2x}}{\cos(x + 3y)} - 1 \right]$$

4. Find the slope of the curve $y^2 + x^2 = y^4 - 2x$ at point $(-2, 1)$.

10 points

$$2yy' + 2x = 4y^3y' - 2$$

$$2(1)y' + 2(-2) = 4(1)^3y' - 2$$

$$2y' - 4 = 4y' - 2$$

$$-2 = 2y'$$

$$y' = -1$$