Name Solutions

Math 141A	Spring 2014
Quiz Five March 14, 2014	

DIRECTIONS: Allow yourself no more than 30 minutes to complete this quiz. **No calculators.** This quiz is given under conditions of the ***Luther College Honor Code.*** You are expected to uphold the highest standards of academic integrity, and you are expected to demand the same from fellow students.

1. Verify that the point (2,3) is on the curved defined by the equation $x^2 + xy - y^2 = 1$ and give equations for the lines that are (a) tangent and (b) normal to the curve at (2,3) [10 points]

Verification: $(2)^2 + (2\times3) - (3)^2 = 4 + 6 - 9 = 1$ ✓

Find $\frac{dy}{dx}$: $2x + y + xy' - 2yy' = 0$

$2x + y = (2y - x)y' \Rightarrow y' = \frac{2x+y}{2y-x}$

$y'(2,3) = \frac{2(2)+3}{2(3)-2} = \boxed{\frac{7}{4}}$

tangent line: $y - 3 = \frac{7}{4}(x-2)$

normal line: $y - 3 = -\frac{4}{7}(x-2)$

2. Find the value of dy/dx for $x \sin 2y = y \cos 2x$ at the point $(\pi/4, \pi/2)$. [10 points]

$(x\sin 2y)' = (y\cos 2x)'$

$\sin 2y + x\cos 2y \cdot (2y)' = y'\cos 2x + y(-\sin 2x)\cdot(2x)'$

$\sin 2y + 2xy'\cos 2y = y'\cos 2x - 2y\sin 2x$

Sub $x = \pi/4$ $y = \pi/2$

$\sin\left(2\frac{\pi}{2}\right) + 2\frac{\pi}{4}y'\cos\left(2\frac{\pi}{2}\right) = y'\cos\left(2\cdot\frac{\pi}{4}\right) - 2\frac{\pi}{2}\sin\left(2\frac{\pi}{4}\right)$

$0 + \frac{\pi}{2}y'(-1) = y'\cdot 0 - \pi$

$-y' = -\pi\left(\frac{2}{\pi}\right) \Rightarrow \boxed{y' = 2}$

3. Let $f(x) = x^3 - 2x^2 + 3$, $x \geq 1$. Find the value of df^{-1}/dx at $x = 3$. 5 points

Use $(f^{-1})'(f(a)) = \dfrac{1}{f'(a)}$ here $f(a) = 3$

or $a^3 - 2a^2 + 3 = 3$ (guess and check) $a = 2$

$a^3 = 2a^2$ (or)

$f'(x) = 3x^2 - 4x$ so $f'(2) = 3 \cdot 4 - 4 \cdot 2 = 12 - 8 = 4$

$$\boxed{(f^{-1})'(3) = \frac{1}{4}}$$

4. Find dy/dt for $y = t^2\sqrt{\ln t}$. 5 points

$$\frac{dy}{dt} = 2t\sqrt{\ln t} + t^2 \cdot \frac{1}{2\sqrt{\ln t}} \cdot \frac{1}{t}$$

$$= \boxed{2t\sqrt{\ln t} + \frac{t}{2\sqrt{\ln t}}}$$

5. Find dy/dx for $y = \tan^{-1}(1 + x^2)$. 5 points

$$\frac{dy}{dx} = \frac{1}{1 + (1+x^2)^2} \cdot (1+x^2)'$$

$$= \boxed{\frac{1}{1 + (1+x^2)^2} \cdot 2x}$$

6. Find dy/dx for $y = \arcsin(e^{-2x})$. 5 points

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - (e^{-2x})^2}} \cdot (e^{-2x})'$$

$$= \boxed{\frac{1}{\sqrt{1 - e^{-4x}}} \cdot e^{-2x} \cdot -2}$$