Name Solutions

Math 141A	Spring 2014
Quiz Six April 23, 2014	

DIRECTIONS: Allow yourself no more than 30 minutes to complete this quiz. **No calculators.** This quiz is given under conditions of the ***Luther College Honor Code***. You are expected to uphold the highest standards of academic integrity, and you are expected to demand the same from fellow students.

1. Find the following indefinite integrals. [15 points]

(a) $\int \left(\frac{-1}{x^3} + x - 1\right) dx$ $= \int (-x^{-3} + x - 1)\, dx$

$= -\frac{x^{-2}}{-2} + \frac{x^2}{2} - x + C$

(b) $\int \left(\cos\left(\frac{\pi x}{2}\right) + \sin x\right) dx$ $= \frac{2}{\pi}\sin\left(\frac{\pi}{2}x\right) - \cos x + C$

(c) $\int \frac{2+\sqrt{t}}{t^2}\, dt$ $= \int \frac{2}{t^2}\, dt + \int \frac{\sqrt{t}}{t^2}\, dt$

$= \int 2t^{-2}\, dt + \int t^{-3/2}\, dt$

$= \frac{2t^{-1}}{-1} + \frac{-2}{1}t^{-1/2} + C$

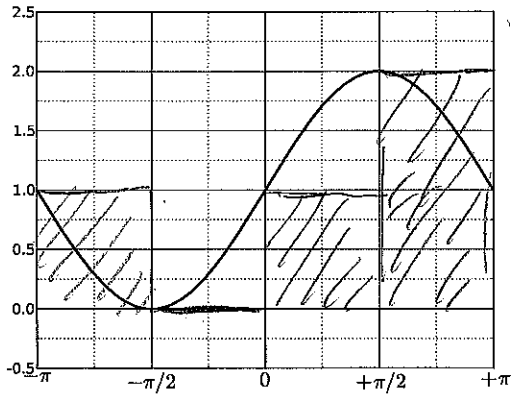
2. Solve the initial value problem $\frac{dy}{dx} = 1 - x$, $y(1) = 0$. [5 points]

$y(x) = x - \frac{x^2}{2} + C$

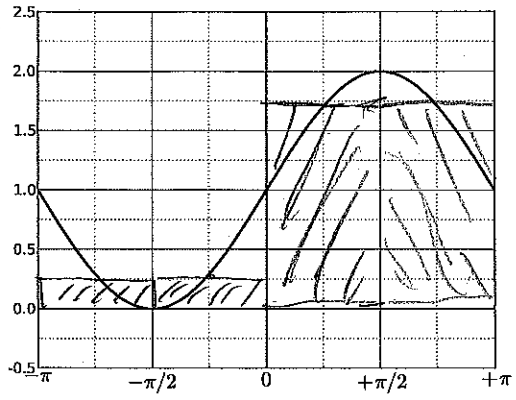
$y(1) = 0 \Rightarrow 1 - \frac{1}{2} + C = 0 \Rightarrow C = -\frac{1}{2}$

$y(x) = x - \frac{x^2}{2} - \frac{1}{2}$

3. The graph shown in Figures 1(a) and 1(b) is that of $f(x) = \sin x + 1$ on the interval $[-\pi, \pi]$.



(a) Left Sum.



(b) Midpoint Sum.

Figure 1: Graphs of $f(x)$.

(a) Sketch the rectangles associated with a left sum of 4 equal subintervals of the interval $[-\pi, \pi]$ on Figure 1(a). Next, write out the sum showing each of the four terms. $\Delta x = \frac{\pi - -\pi}{4} = \frac{\pi}{2}$ 10 points

$$LS = \left[f(-\pi) + f(-\tfrac{\pi}{2}) + f(0) + f(\tfrac{\pi}{2}) \right] \frac{\pi}{2}$$

$$= \left[1 + 0 + 1 + 2 \right] \frac{\pi}{2}$$

$$= \boxed{2\pi}$$

(b) Sketch the rectangles associated with the midpoint sum of 4 equal subintervals of the interval $[-\pi, \pi]$ on Figure 1(b). Next, write out the sum showing each of the four terms. 10 points

$$MS = \left[f(-\tfrac{3\pi}{4}) + f(-\tfrac{\pi}{4}) + f(\tfrac{\pi}{4}) + f(\tfrac{3\pi}{4}) \right] \frac{\pi}{2}$$

$$= \left[-\frac{\sqrt{2}}{2} + 1 + -\frac{\sqrt{2}}{2} + 1 + 1 + \frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2} \right] \frac{\pi}{2}$$

$$= 4 \cdot \frac{\pi}{2}$$

$$= \boxed{2\pi}$$