Chapter One Examples

1. Determine the average speed of the molecules in a parcel of air at the temperature of 70° F.

   Solution: First, convert the temperature to K, T = 273 + (5/9)(70-32) = 294.1. Next, use the equation $T = \alpha m_W v^2$ and solve for speed v: $v^2 = \dfrac{T}{\alpha m_W} \Rightarrow v = \sqrt{\dfrac{T}{\alpha m_W}} = \sqrt{\dfrac{294.1}{4.0 \times 10^{-5} \cdot 28.9}} = 504.3$ m/sec

2. Calculate the surface pressure if the surface density is $\rho_s = 1.22$ g/L and the surface temperature is T_s=60° F.

   Solution: Use the Ideal Gas Law : $P_s = 2.87 \rho_s T_s \Rightarrow P_s = 2.87 \times 1.22 \times (\frac{5}{9}(60-32)+273) \Rightarrow P_s = 1010.3$ mb

3. Given that the surface temperature is 90° F, find the temperature at the elevation of 5 km assuming a standard lapse rate.

   Solution: The surface temperature is 32.2° C. Use the formula $T(z) = T_s - \dfrac{6.5}{1000} z$ which gives

   $T(5000) = 32.2 - \dfrac{6.5}{1000} 5000 = -0.3°\ \text{C} = 31.5°\ \text{F}$

4. Suppose the surface temperature is 25 C, and the lapse rate within the first 2000 m is 5 ° C/1000 m, while the lapse rate in the interval between 2000 m and 5000 m is 4 ° C/1000 m. Determine the temperature at (a) 1000 m and (b) 4000 m.

   Solution:

   (a) $T(1000) = 25 - \frac{5}{1000} \times 1000 = 20$

   (b) $T(4000) = 25 - \frac{5}{1000} \times 2000 - \frac{4}{1000} \times 2000 = 25 - 10 - 8 = 7$ C

5. Find the density in g/L of a parcel of air at the elevation of one mile assuming a standard atmosphere and a surface density of 1.22 g/L.

   Solution: Use the formula $\rho(z) = \rho_s e^{-z/8000}$. Must first convert one mile into meters: $z = 1$ mile $\times$ 5,280 feet/mile $\times$ 0.3049 meters/foot $= 1609.8$ m. Then $\rho(1609.8) = 1.22 e^{-1609.8/8000} = 0.997$ g/L

6. Suppose the surface pressure is $P_s = 1000$ mb. Determine the elevation at which the pressure is 500 mb assuming a standard atmosphere.

   Solution: In a standard atmosphere we have $P(z) = P_s e^{-z/8000}$. In this example we are solving for elevation z. Therefore $500 = 1000 e^{-z/8000}$ which means $e^{-z/8000} = 0.5$. Take the natural log of both sides to get $-z/8000 = \ln 0.5$, so that $z = -8000 \times -0.6931 = 5545$ m.

7. Determine the mass of oxygen in a two-liter parcel of air at an elevation of 2000 m. Assume a standard atmosphere with a surface density of 1.22 g/L.

   Solution: Determine the mass, in grams, of a one-liter parcel by determining its density at an elevation of 2000 m: $\rho(2000) = 1.22 e^{-2000/8000} = 0.95$ g/L. The percentage of the mass of oxygen in the parcel is approximately the same as the percentage of volume of oxygen for any parcel. Therefore 21% of the parcel mass is oxygen, so in the case of a two-liter parcel we have $2 \times 0.21 \times 0.95 = 0.399$ g of oxygen.

8. The pressure at the top of a mountain is measured to be 800 mb. If the surface temperature at the base of the mountain is 70° F, determine the elevation of the mountain top.

   Solution: Eventually, we will use $P(z) = P_s e^{-z/8000}$ in its log form to determine the elevation z. First, we need the surface pressure which is calculated using the surface temperature and density and the Ideal Gas Law. $P_s = 2.87 \times 1.22 \times (\frac{5}{9}(70 - 32) + 273) = 1029.8$ mb. The result for z is: $z = -8000 \ln(800/1029.8) = 2020$ m

9. For a standard atmosphere with a surface pressure of 1000 mb, determine the elevation at which 75% of the atmosphere's mass is above this point.

   Solution: Pressure is just the mass per unit area above the given level. So we are looking for the point at which the pressure is 75% of the surface pressure. Using $P(z) = P_s e^{-z/8000}$, this means we want to find z such that $75\% \times 1000 = 1000e^{-z/8000}$ which means $750 = 1000e^{-z/8000}$. Solving for z using the natural log gives $z = -8000 \ln(750/1000) = 2301$ m

10. The potential temperature θ of a parcel at 750 mb is 282K. Determine the temperature of the parcel in degrees Celsius.

    Solution: If not otherwise specified, the reference pressure level is 1000 mb. The relationship between temperature T and potential temperature θ is $\theta = T\left(\frac{1000}{P}\right)^{0.2859}$. Solving for T in terms of θ gives $T = \theta\left(\frac{1000}{P}\right)^{-0.2859} = \theta\left(\frac{P}{1000}\right)^{0.2859}$. Subbing values for $\theta = 282$ and $P = 750$ gives $T = 282(0.75)^{0.2859} = 282 \cdot 0.9210 = 259.7$K. The temperature in Celsius is 259.7-273 = -13.26 C.

11. An atmospheric parcel has a temperature of -25 C and a potential temperature of 322 K. Estimate the parcel's elevation if the reference (surface) pressure is 1000 mb.

    Solution: Knowing the temperature and potential temperature of the parcel will allow us to determine the pressure of the parcel. From the parcel's pressure, we should be able to determine the approximate elevation. We have $322 = (-25 + 273)\left(\frac{1000}{P}\right)^{0.2859}$. We need to solve this expression for P. Some of the intermediate steps are shown :

    $$\frac{322}{248} = \left(\frac{1000}{P}\right)^{0.2859} \Rightarrow \left(\frac{322}{248}\right)^{1/0.2859} = \frac{1000}{P} \Rightarrow 2.493 = \frac{1000}{P} \Rightarrow P \approx 400 \text{ mb}$$

    Next, we determine z :

    $$400 = 1000e^{-z/8000} \Rightarrow \frac{z}{8000} = \ln\left(\frac{400}{1000}\right) \Rightarrow z = -8000(-0.9163) \approx 7330 \text{ m}$$

12. Calculate the total mass of air in a column, extending from the surface of the earth to the "top" of the atmosphere, if the surface pressure is 1000 mb and the column has a uniform width of 1 square meter.

    Solution: Pressure is a measure of force per area, and force is the product of mass and the acceleration due to gravity (9.8 m/sec^2).

    $$1000 \text{ mb} = 1000 \times 100 \text{ N/m}^2 = \frac{100,000}{1} = \frac{\text{mass of air } \times 9.8}{1} \Rightarrow \text{mass} = 100,000/9.8 \approx 10,200 \text{ kg}$$

    Note: The first step in the solution is changing pressure in mb to pressure in Pascals, or Newtons/meter-squared. The weight of this mass, in pounds is $2.2\frac{\text{lbs}}{\text{kg}} \times 10,200 \approx 22,440$ pounds.

13. A column of air with a base area of 1 square meter has an average mixing ratio of 10 g/kg. Precipitation dynamics within the column result in 60% of the water vapor precipitating in the form of rain. The column from which the precipitation falls extends from 1000 m to 4000 m in elevation. Determine the depth of the precipitation in centimeters.

    Solution: First, estimate the total mass of the dry air in the column (neglect the mass of water vapor - it represents a very small portion of the total mass). Use the result from example 12 to determine the mass of atmosphere above 1000 meters, and the mass of atmosphere above 4000 meters. The mass between these elevations is just the difference in these values. The mass above 1000 meters = $10,200e^{-1000/8000} = 9005$ kg. The mass of atmosphere above 4000 meters = $10,200e^{-4000/8000} = 6189$ kg. Therefore, the mass between these respective elevations is 9005 kg - 6189 kg = 2816 kg.

    Next, use the mixing ratio to determine the mass of water vapor in the column section. Mass of H_2O = 2816 kg × 10 g/kg = 28,160 g. Now, 60% precipitates out, so the total mass of rain water is 0.60 × 28,160 = 16,896 g. Now, 1 g of H_2O has a volume of 1 cubic cm (cc = cm^3), so the total volume of rain is 16,896 cc. Our column has a base area of 1 m^2 = 100 × 100 cm^2 = 10,000 cm^2.

    Finally, the depth of water is the volume of water divided by the base area = 16,896 cm^3 / 10,000 cm^2 = 1.69 cm. This depth in inches = 1.69 cm / (2.54 cm / in) ≈ 0.66 inches.