State Reduction?

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Many writings about quantum measurement posit a *universal* state reduction: *every* quantum measurement is accompanied by a state reduction. The supplementary material (next page) provides many examples. However, there are measurements without a state reduction. So authors and teachers should refrain from stating that state reduction is universal.

To discuss this we need two definitions: measurement and state reduction. They are implicitly, and sometimes explicitly, used by all the authors in the supplementary material, with one instructive exception – Einstein.

Measurement. Let $\mathcal{Q}$ be a quantum system with a finite dimensional state space S. Let A be a Hermitian operator on S representing an observable $\mathcal{A}$ of $\mathcal{Q}$. The operator A has an orthonormal basis of eigenvectors $\{\mathbf{a}_i\}$ with eigenvalues $\{a_i\}$, respectively. A *measurement* of $\mathcal{A}$ creates a *number* on a macroscopic measuring apparatus. The number is an eigenvalue a_j of A, with probabilities of the a_i given by Born's rule.

The postmeasurement state of $\mathcal{Q}$ is the matter of interest in this letter.

State reduction. Measure $\mathcal{A}$ on state $\mathbf{s}$ of S. Suppose that the result is a_j. Then a *state reduction* occurs if the immediate postmeasurement state of $\mathcal{Q}$ is the projection of $\mathbf{s}$ onto the eigenspace of a_j, normalized (Lüders rule). In the special case where a_j is nondegenerate, distinct from all of the other a_i, this becomes simply $\mathbf{a}_j$.

Repeatability is equivalent to state reduction: an immediate subsequent measurement of the same observable A gives the same result a_j. This formulation of state reduction is often used.

There are trivial counterexamples to universal state reduction. An example is the measurement of photon polarization where the photon is destroyed in a detector. This is a measurement, but the photon is not left in *any* state. It is not unusual that $\mathcal{Q}$ is destroyed in a measurement. This alone is sufficient to reject universal state reduction.

Even if $\mathcal{Q}$ survives, there need not be a state reduction. John Bell is the author of the famous Bell's inequality and expert accelerator designer at CERN. He and Michael Nauenberg gave an example:

> Suppose for example the momentum of a neutron is measured by observing a recoil proton. The momentum of the neutron is altered in the process, and in a head-on collision actually reduced to zero. The subsequent state of the neutron is by no means a combination (the spin here provides the degeneracy) of states with the observed momentum. How then is one to know whether a given measurement is [accompanied by a state reduction], or not? Clearly, one must investigate the physics of the process.[1]

Momentum has a continuous spectrum, so this might not seem relevant to our finite dimensional state space S. But a detector has a finite resolution, so in a real experiment measurement values must be discretized by binning.

Google Scholar lists 39 papers which cite the Bell-Nauenberg paper. I read their titles and abstracts and looked at many of the papers. None criticize the design or interpretation of the experiment.

Since a state reduction need not occur, it is impossible to prove a state reduction without additional assumptions about the physics of the measuring process, as Bell and Nauenberg write. Nevertheless, the supplementary material shows that a universal state reduction is widely accepted without additional assumptions, starting already with the founders of quantum mechanics.

The supplementary material reviews some of the history of the idea of universal state reduction. It cites the views of three categories of writers: *The Founders*, *Scholars of the Foundations*, and *Popular Textbooks.* The founders, with one exception, and all the textbooks, accept universal state reduction. The scholars of the foundations, with one exception, do not.

Online Supplement to "State Reduction?"

The Founders

Dirac. Paul Dirac writes of measurement "When we make an observation we measure some dynamical variable. It is obvious physically that the result of such a measurement must always be a real number, so we should expect that any dynamical variable that we can measure must be a real dynamical variable."[2] He then asserts universal state reduction: "a measurement *always* causes the system to jump into an eigenstate of the dynamical variable that is being measured" (emphasis added).[3]

Pauli. Wolfgang Pauli writes: "The method of the energy discussed till now has the property that a repetition of measurement gives the same value as in the first measurement. We shall call such measurements as measurements of the first kind. On the other hand it can also happen that the result of a repeated measurement is not the same as that of the first measurement. Such measurements, we call the measurements of the second kind."[4] (I have left out parts of Pauli's rather verbose statement.) So Pauli recognizes that a state reduction need not occur in a measurement.

Einstein. Henry Margenau quotes from a personal letter from Albert Einstein advocating state reduction:

> The present form of quantum mechanics is adjusted to the following postulate, which seems inevitable in view of the facts of experience: If a measurement performed upon a system yields a value m, then the same measurement performed immediately afterwards yields again the value m with certainty. Example: If a quantum of light [a photon] has passed a polarizer P_1, then I know with certainty that it will also pass a second polarizer P_2 which has its orientation parallel to the first.[5]

So for Einstein the passage through polarizer P_1 is a "measurement" (his word) with a state reduction "with certainty". Margenau counters with the concept of state *preparation*, different from *measurement*. It was already used by Dirac.[6] The passage of the photon through polarizer P_1 is not a measurement as defined in the Letter; there is no "number on a measuring apparatus." Instead, the passage is better called a *preparation*: it *prepares* the photon in a particular state, which then can be *measured*. Conceptually different actions should have different names. *First* prepare a state, *then* measure it. The different actions are represented by different parts of the quantum formalism: states by vectors in a Hilbert space and measurements by self-adjoint operators on the space.

According to Nobel Prize winning experimentalist Willis Lamb: "Although some authors confuse preparation of a state and measurement, these concepts are logically and physically very different."[7]

The concept of preparation is widely used today in some areas of quantum physics. A prominent example is quantum information theory. In the classic book *Quantum Computation and Quantum Information*,[8] and the book *Probabilistic and Statistical Aspects of Quantum Theory*,[9] "prepare" and "preparation" are part of the basic vocabulary, as are the conceptually different "measure" and "measurement". Another example comes from the paper *Quantum state preparation of normal distributions using matrix product states*. Its abstract begins with "State preparation is a necessary component of many quantum algorithms."[10]

von Neumann. According to Max Jammer, "von Neumann's conception of the measurement problem became the framework of almost all subsequent theories of measurement."[11] This includes his treatment of state reduction.

von Neumann on measurement: "In a measurement we cannot observe the system $\mathbf{S}$ by itself, but must rather investigate the system $\mathbf{S} + \mathbf{M}$ in order to obtain (*numerically*) its interaction with the measuring apparatus $\mathbf{M}$ (emphasis added)."[12] This process "is in general irreversible".[13]

von Neumann analyzed the Compton-Simons experiment of scattering of photons by electrons. He writes: "We used the following principle, abstracted from the result of the experiment: if the physical quantity $\mathcal{R}$ is measured twice in prompt succession in a system $\mathbf{S}$ then we get the same value each time.[14] So von Neumann "abstracted" *universal* state reduction from a *single* experiment in which a state reduction does occur.

Wigner. After describing the continuous Schrödinger evolution of the state vector, Eugene Wigner writes: "The state vector changes, however, also discontinuously, according to probability laws, if a measurement is carried out on the system. This second type of change is often called the reduction of the wavefunction."[15] Wigner allows no exceptions.

Scholars of the Foundations

Reichenbach. Hans Reichenbach first defines measurement in classical mechanics: "A measurement is a physical operation which furnishes a determinate *numerical* result and which in an immediate repetition furnishes the same result (emphasis added)." Reichenbach then writes: "It is an advantage of the given definition that it can be applied also in quantum mechanics".[16] He thus mandates state reduction in quantum measurements by definition. By his definition, neither the photon polarization detection nor Bell-Nauenberg's experiment is a measurement, as there is no state reduction.

Bell-Nauenberg. As shown in the Letter, Bell and Nauenberg's experiment is a measurement with no state reduction.

Margenau. Henry Margenau does not accept universal state reduction:

> We find no general substantiation of the projection postulate, nor of the "reduction of the wave packet" if the latter is taken to mean the automatic establishment of a new ϕ after the measurement.[17]

Jauch. Josef Jauch acknowledges that not all measurements lead to a state reduction:

> We shall call a measurement which will give the same value when immediately repeated a measurement of the first kind. The second example [similar to Bell-Nauenberg's] is then a measurement of the second kind. From now on we shall be primarily concerned with measurements of the first kind. They are easier to discuss, yet they exhibit the characteristic quantum features which we want to explore here.[18]

This seems to me to be a reasonable approach.

d'Espagnat. For Bernhard d'Espagnat state reduction is not only not the rule, but the exception. After describing a measurement with a state reduction, he writes: "Such a measurement will be called an *ideal* one or a *measurement of first kind*. All other measurements are measurements of the *second kind*." And: "When a quantum theory of the measurement process is developed, the important facts that the initial state of the instrument is not known exactly, and that *most measurements are nonideal* . . . , should be kept carefully in mind (emphasis added)."[19].

Popular Textbooks

Mandl. Franz Mandl accepts universal state reduction in his Postulate V.[20]

Cohen-Tannoudji, Diu, and Laloe. In the section *REDUCTION OF THE WAVE PACKET*, the authors write:

> Let us first consider the case where the measurement of $\mathcal{A}$ yields a simple [i.e., nondegenerate] eigenvalue a_n of the observable A. We then postulate that the state of the system immediately after this measurement is the eigenvector $|\mathbf{u}_n\rangle$ associated with a_n.[21]

When the eigenvalue a_n given by the measurement is degenerate, the authors postulate that the state of the system immediately after the measurement is given by Lüders rule. So they accept universal state reduction.

Sakurai. Jun John Sakurai accepts universal state reduction by simply quoting Dirac's statement cited above.[22]

Griffiths. David Griffiths accepts universal state reduction by assuming its equivalent as a fact:

> Moreover, to account for the fact that an immediately repeated measurement yields the same result, we are forced to assume that the act of measurement **collapses** the wave function.[23]

Zettili. Nouredine Zettili accepts universal state reduction: "The act of measuring A changes the state of the system from ψ to one of the eigenstates ψ_n of the operator $\hat{A}$, and the result obtained is the eigenvalue a_n."[24]

Longair. Malcolm Longair writes: "As a result of the measurement of A in which the result a is obtained, the state of the system is changed to the corresponding eigenstate **a**. . . . This process of forcing the system into one of the possible state functions is referred to as the *collapse* or *reduction* of the wavefunction."[25]

Susskind and Friedman. Leonard Susskind and Art Friedman write: "During an experiment the state of a system jumps unpredictably to an eigenstate of the observable that was measured. This phenomenon is called *the collapse of the wave function*."[26]

Rae and Napolitano. Alastair Rae and Jim Napolitano write:

> Immediately after such a measurement, the wave function of the system is identical to the eigenfunction corresponding to the eigenvalue obtained as a result of the measurement. . . . This transformation of the wavefunction from its original form to one equal to that of the eigenfunction is often described as "the collapse of the wavefunction.[27]

[1] John S. Bell and Michael Nauenberg, "The moral aspect of quantum mechanics", in *Speakable and Unspeakable in Quantum Mechanics*, (Cambridge University Press, Cambridge, 1987), p. 23.
[2] Paul Adrien Maurice Dirac, *Principles of quantum mechanics*, 3rd Ed. (The Clarendon Press, Oxford, 1947), p. 34.
[3] ibid., p. 36.
[4] Wolfgang Pauli, *The General Principles of Quantum Mechanics* (Springer-Verlag, Berlin, 1980), p. 75. This is a translation of *Die allgemeinen Prinzipien der Wellenmechanik*, published in 1933.
[5] Henry Margenau, "Philosophical Problems concerning the Meaning of Measurement in Physics", Phil. Sci. 25 (1), pp. 3–33 (1958).
[6] Dirac, ibid., pp. 11–12.
[7] Willis Lamb Jr., "An operational interpretation of nonrelativistic quantum mechanics", Physics Today 22 (4), pp. 23-28 (1969).
[8] Michael A. Nielsen and Isaac L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, New York, 2000).
[9] Alexander Holevo, *Probabilistic and Statistical Aspects of Quantum Theory* (Elsevier Science, Amsterdam, 1982), p. 6.
[10] Jason Iaconis, Sonika Johri, & Elton Yechao Zhu, "Quantum state preparation of normal distributions using matrix product states", npj Quantum Information 10 (1), pp. 15-26 (2024)
[11] Max Jammer, *The Philosophy of Quantum Mechanics* (Wiley, New York, 1977), p. 475.
[12] John von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Princeton, 2018), p. 230. This is a translation of *Mathematische Grundlagen der Quantenmechanik*, (Springer, Berlin, 1932).
[13] ibid., p. 234.
[14] ibid., pp. 218–219.
[15] Eugene Wigner, "The Problem of Measurement", Am. J. Phys. 31 (1), pp. 6–15 (1963).
[16] Hans Reichenbach, *Philosophic Foundations of Quantum Mechanics* (University of California Press, Berkeley, 1944), pp. 95-96.
[17] Henry Margenau, "Measurements and Quantum States: Part II", Phil. Sci. 30 (2), p. 143 (1963).
[18] Josef Jauch, *Foundations of Quantum Mechanics* (Addison-Wesley, Reading MA, 1968), p. 165.
[19] Bernard d'Espagnat, *Conceptual Foundations of Quantum Mechanics*, 2nd Ed. (Perseus Books, Reading, Massachusetts, 1976), pp. 18, 159.
[20] Franz Mandl, *Quantum Mechanics*, 2nd Ed. (Butterworths, London, 1957), p. 69.
[21] Claude Cohen-Tannoudji, Bernard Diu, and Frank Laloe, *Quantum Mechanics, Volume 1* (Wiley, New York, 1991), p. 220.
[22] Jun John Sakurai, *Modern Quantum Mechanics*, Revised Ed. (Addison-Wesley, Reading, MA, 1994), p. 23.
[23] David J. Griffiths, *Introduction to quantum mechanics* (Prentice Hall, Englewood Cliffs, N.J., 1995), p. 374.
[24] Neuroedine Zettili, *Quantum Mechanics: Concepts and Applications*, 2nd Edition (Chichester, U.K, 2009), p. 173.
[25] Malcolm Longair, *Quantum Concepts in Physics*, (Cambridge University Press, Cambridge, U.K., 2013), p. 367.
[26] Leonard Susskind and Art Friedman, *Quantum Mechanics: The Theoretical Minimum* (Basic Books, New York, 2014), p. 201.
[27] Alastair Rae and Jim Napolitano, *Quantum Mechanics*, 6th Ed., (Taylor and Francis Group, Boca Raton, Florida, 2016), p. 142.
More generally, states are represented by density operators and measurements by positive operator-valued measure (POVM)